Open-Pit Mine Production Optimization: A Review of Models and Algorithms

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Abstract:

Plan and schedule based on which material of an open pit mine are being handled includes two main steps: static scheduling and real-time dispatching. Earlier is done statically at the beginning of the shift and by the time status of mine changes significantly. Along with, the later one is a dynamic decision making procedure running all over the shift. Open pit mines operation costs highly depends on material handling. 50 to 60 percent of operating costs in an open pit mine is spent on digging and transporting the material. So, reducing a small portion of it will save billions of dollars in a large open pit mine. Generally there are two ways to reduce this cost: transferring higher amount of material in each payload and optimizing both static and dynamic operation schedules. To achieve the later goal different algorithms have been developed since late 1960s. This paper shows that from applicability point of view there are two major groups of algorithms: a) the group are being widely used in the mining projects, and b) the group of algorithms developed in academy. The paper first discusses main industrial algorithm, then after reviews well-known academically developed algorithms. Strengths and weaknesses of the algorithms are discussed and suggestions for the future researches are presented.

1. Introduction

Mining projects and more especially surface mines are known as high cost expenditures that need millions of dollars or in the large ones billions of dollars to be expend on them in both capital and operating parts. Material handling procedure as the main consumer of the operating cost plays a critical role in the mining projects decision making procedure. A large portion of total mining costs in an open pit mine must be allocated to excavating and transporting the excavated materials from the mining faces to different destinations out of the pit rim. As it is believed by many researchers, 50% of operating costs in open pit mines (Alarie and Gamache, 2002) and even in some cases especially in large open pit mines up to 60% of the operation costs is to be spent on material handling (Alarie and Gamache, 2002; Oraee K. and Goodarzi A., 2007; Akbari et al., 2009; Ahangaran et al., 2012; Upadhyay and Askari-Nasab, 2015). So, improving the transportation operation and subsequently decreasing expenses of this part of the operation even by 2 or 3 percent will save stockholders a huge amount of money. There are two principle way to improve material transportation efficiency in open pit mines. The first way is to implement large size trucks in the truck fleet with the capacity of transporting more material in each payload, the point current truck

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manufacturers have been reaching to the maturity. The second principle way to improve the transportation operation reduce cost per ton of material transported is to implement operations research techniques to enhance productivity of the operation. Although as Alarie and Gamache (2002) considers there is a single stage approach like the one was presented by (Hauck, 1973) which implement a continuous algorithm to maximize productivity of the operation and send trucks to the destination in a way that minimize deviation from the production target simultaneously, there is a multistage approach of the open-pit operation optimization that is of the most interest under which the problem is divided into two sub problems. In the first sub problem a static scheduling algorithm is implemented to determine the optimal loaders configuration over the mining faces as well optimum production rate for each route connecting loading points to discharge points and also allocation of truck resources to meet production target. This stage called upper stage runs at the beginning of the shift and when the mine status changes. As the lower stage a dynamic algorithm mostly based on assignment problem or rarely based on transportation problem assigns the trucks to a proper destination by the time they asks for a destination in the way that minimize deviation from the production target.

There are two basic categories for open pit mines' operation optimization. Industrial groups who present the software packages to the mining projects without disclosing algorithms behind the software; the academic groups who, although, disclose all logics behind the algorithms never implement the methods world widely.

2. Definition of the Operational Planning Problem in Surface Mines

A mine's production schedule include three time range plans: 1 – Long-term (20 – 30 years length, describe feasibility of the mining adventure and cash flow distribution, and is the major input of the medium- and short-term plan); 2 – Medium-term (1 – 5 years length, provides more detail information for extraction of mining areas specifically, presents more information about fleet expansion or equipment replacement); 3 – Short-term (1 – 12 months, detailed information about faces to be extracted and feeds to be sent to the plant) (Osanloo et al., 2008). Short-term schedule itself is broken down to operational plans. Operational plan is the shift base stage of open pit mine production scheduling which covers dynamic real-time decision making procedure in surface mine operation that includes: finding the shortest paths between loading and discharge points, operation optimization that means finding optimum productivity rate of each route and allocate truck payload to each route in a way that cover the production target (upper stage), and dynamic truck assignment (lower stage) which is illustrated in Fig 1. Indeed, the operational planning tasks challenge with the equipment allocation problem tries to make decision on allocation and dispatching of the trucks based on the routes capacity and equipment capabilities (Newman et al., 2010).

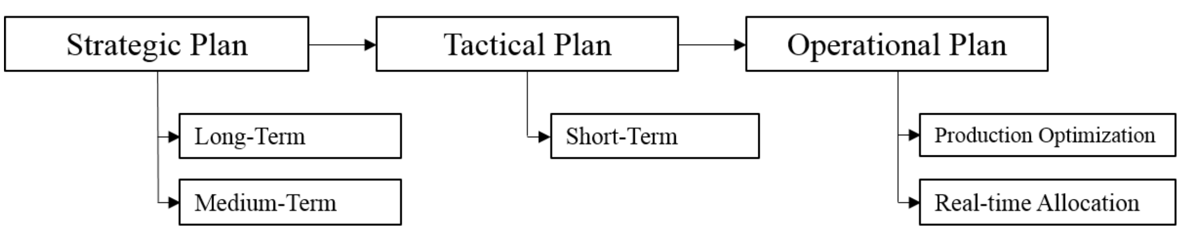

Fig 1. Stages of Making Decisions in Mine Production Scheduling (Upadhyay and Askari-Nasab, 2015).

Operation plan as it is shown in Fig. 2 runs all over the mine life. As it is illustrated it accepts schedule of each period from the short-term schedule and available mining faces. Then it runs until the end of the shift or the time mine status changes.

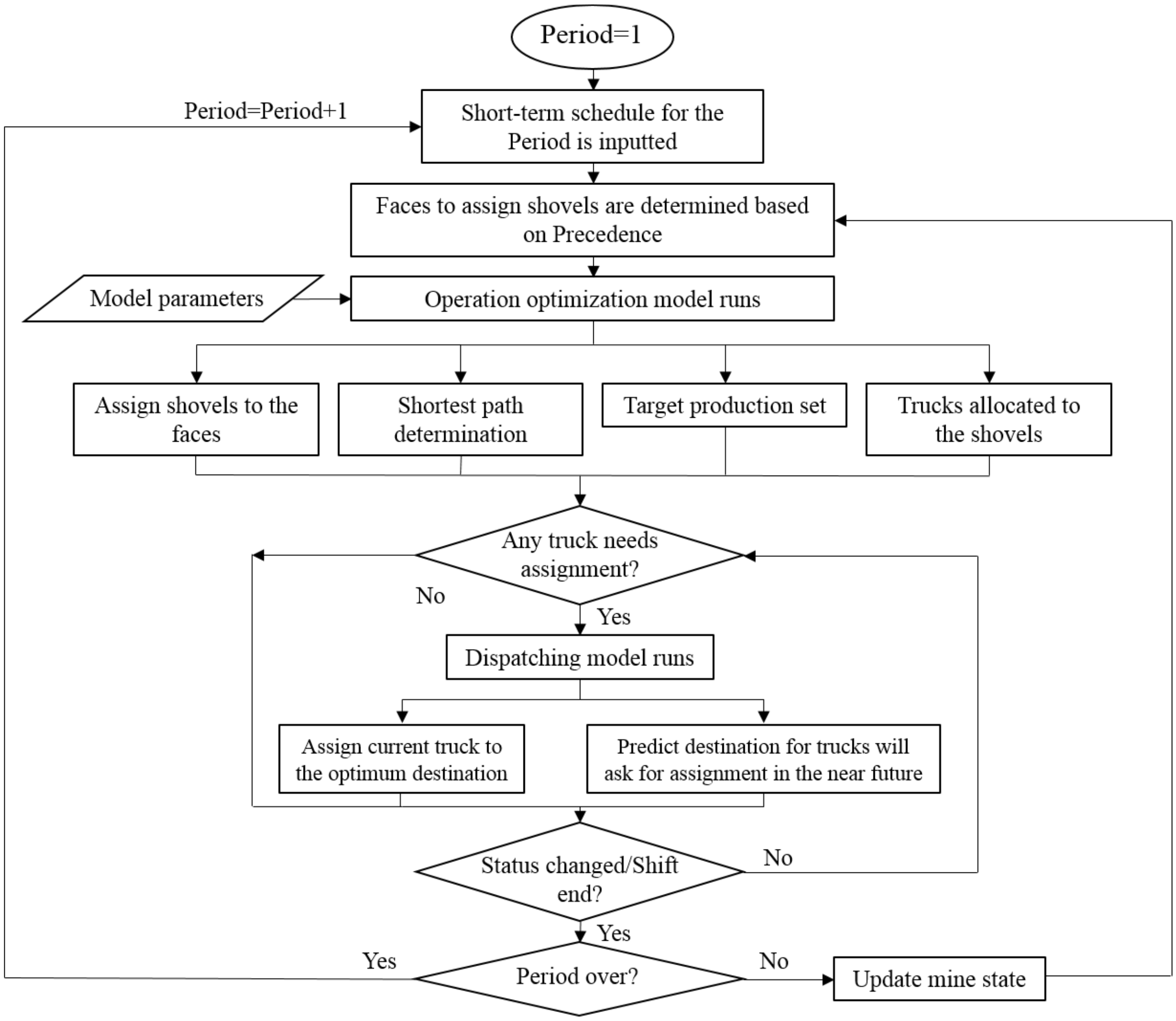

Fig 2. schematic illustration of Open Pit mine operational plan (Upadhyay and Askari-Nasab, 2015).

Researchers in both academic institutes and industrial sectors have been trying to improve the algorithms of open pit mine operational plans to reduce the cost of the project or increase the production level. Herein, we reviewed both sides of the development including industrial algorithms and academically improved ones, though for the former one there is not sufficient revealed algorithms.

3. Fixed Truck Allocation

In this type of mine operation at the beginning of each shift a group of trucks are locked to each transportation route. The trucks allocated to the paths are to work on the same path over the shift period based on some criteria such as production requirement, availability of the trucks in the fleet, and so on (Lizotte and Bonates, 1987; Lizotte et al., 1987). The paths to which trucks have been allocated to will not change until a shovel breaks down or a critical event happens. Some efforts to modify this method has been seen in the literature. Firstly, Bogert (1964) suggested using of radio communication between equipment operators and mine control center. Late 1970s Mueller (1977) introduced implementation of the dispatching boards installed in the control center. This method of operation scheduling is the least productive method and From Kolonja and Mutmansky (1993) to Hashemi and Sattarvand (2015) it has been always being used as the base method to study other algorithms and approaches.

4. Flexible Truck Allocation

In this type of mine operation scheduling a portion of available trucks of the fleet are assigned to a specific working shovel at the beginning of the shift. But these trucks instead of being in the service of only a single shovel or a single route whole the shift, they ask for a new assignment each and every time they loaded the material at the loader or discharge it in a dump area. This method of taking equipment into the work, researcher claim that improves productivity of the operation with a high percentage. Olson et al. (1993) enclosed a 13% increase in the production Bougainville Copper Mine, 10 to 15 % improvement in the productivity of the Barrick Goldstrike Gold mine, 10 % of growth in Iron ore production of LTV steel mining, and 10% increase in the production of the Quintette Coal mine. Furthermore, Hashemi and Sattarvand (2015) in a simulation study of the Sungun Copper Mine's operation showed that by implementing a flexible allocation the productivity of the mine increased by 8% in comparison with the fixed allocation. Also, Kolonja and Mutmansky (1993) study of different types of open pit mine production management heuristics including minimize shovel waiting time (MSWT), minimize shovel saturation (MSS), minimize shovel production requirement (MSPR), minimize truck waiting time (MTWT), minimize truck cycle time (MTCT), and logic behind DISPATCH shows that no matter what type of the flexible truck allocation algorithm be used it always improves productivity in comparison with the fixed allocation (FA) (Table 1).

Table 1: Effect of flexible allocation on open pit mine production (Kolonja and Mutmansky, 1993)

	Percentage difference in production					
System Comparison	13 trucks	Significant difference	16 trucks	Significant difference	18 trucks	Significant difference
FA vs. MSWT	2.97	no	5.67	yes	3.35	yes
FA vs. MSS	3.84	no	5.62	yes	2.89	yes
FA vs. MSPR	2.94	no	4.52	yes	1.33	no
FA vs. MTWT	4.82	yes	6.88	yes	1.47	no
FA vs. MTCT	1.96	no	4.30	yes	1.63	no
FA vs. DISPCH	4.46	yes	7.15	yes	3.15	yes

Since late 1960s number of research have been done in both industries and academies to enhance productivity and reduce cost of mining operation by developing flexible allocation models and algorithms based on different strategies.

4.1. Algorithms Implemented in Industrial Packages

There are many companies across the world providing mine operation management system. From them Mudular mining system with 12% improvement in productivity and accompanying in 200 mines around the world is the leader. Jigsaw with 130 mine is in the second place. However, Wenco by presenting FleetControl claims of 11% improvement in system productivity. They currently have 65 mine sites using their system. CMC introduces Dynamine with a range of productivity improvement of 10% to 15%, Micromine with Pitram system and Caterpillar are the next leader of mine operation management system. Commercial companies who supports mine fleet management do not have willingness of disclosing the logics behind their fleet manager software, though. However, in 1980s and early 1990s Mudular Mining System revealed the models and algorithms based on which DISPATCH mine fleet management system works. Thus, in this section we try to review the algorithms behind DISPATCH from finding the shortest paths to real-time dispatching. Fig 3 and Fig 4 illustrate the procedure DISPATCH goes through to find the solution and the algorithms implements to complete the tasks, respectively.

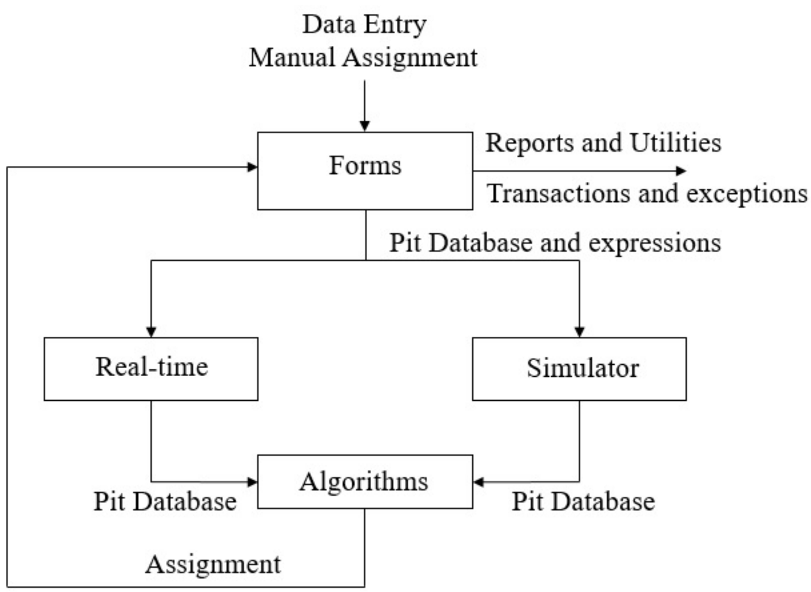

Fig 3. Schematic representation of DISPATCH block diagram (Olson et al., 1993)

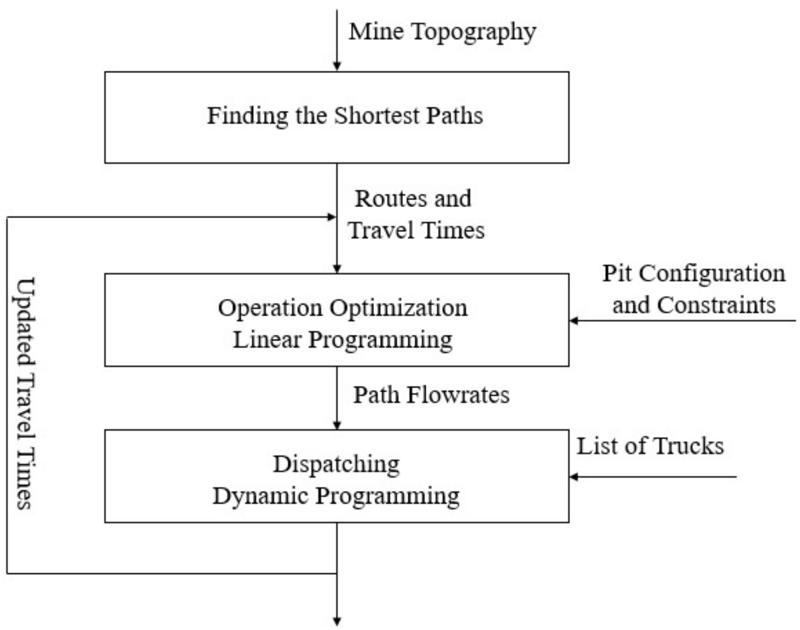

Fig 4. Procedure through with DISPATCH assigns trucks (Olson et al., 1993)

4.1.1. Finding the Shortest Path

In graph theory, the shortest path problem is the problem of finding a path between two vertices (or nodes) in a graph such that the sum of the weights of its constituent edges is minimized. Among different algorithms to find the shortest path in the literature of operations research such as Dijkstra, Bellman – Ford, A* search, Floyd – Warshall, Johnson, Viterbi, and so on, DISPATCH uses Dijkstra's algorithm with the objective of minimizing travel time between each pair of starting and ending points. After solving the shortest path problem in DISPATCH following information is presented to the operation optimization model: 1- total minimum distance and travel time for each specific transport. 2- The nodes truck must passes from to reach destination.

4.1.2. Production Optimization and Truck Allocation

DISPATCH uses linear programming (LP) approach to optimize production target of the time horizon by dividing it into two separated but weakly coupled models from which the first one (Eq. (1)) optimizes total operation including mine sector, plant sector and stockpile, and the second part (Eq. (5)) maximizes the fleet production by minimizing total required volume to be handled. White and Olson (1986) and Olson et al. (1993) describe the model as follow:

$$\min C = \sum_{i=1}^{N_m}(C_m \times Q_i) + C_p \times (P_t - \sum_{i=1}^{N_m+N_s} Q_i) + \sum_{i=1}^{N_s}(C_s \times Q_i) + \sum_{i=1}^{N_m+N_s}\sum_{j=1}^{N_q}(L_j \times C_q \times X_{ij} \times Q_i) \tag{1}$$

Subject to:

$$0 \le Q_i \le R_i \tag{2}$$

$$P_t \ge \sum_{i=1}^{N_m+N_s} Q_i \tag{3}$$

$$X_j L \le X_j A + \sum_{i=1}^{N_m+N_s} \left(X_{ij} - X_j A\right) \times Q_i \times T_c / \left(M_c / SG\right) \le X_j U \tag{4}$$

Where:

C	is functional dimensionless pseudo cost
N_m	is number of shovels at mining faces
C_m	is material transportation pseudo cost (hr/m^3)
Q_i	is material being transported per hour (m^3/hr) that should be determined in the procedure
N_s	is number of shovels working at stockpile
C_s	is stockpile material handling pseudo cost (hr/m^3)
N_q	is number of quality constraints
L_j	is quality director: 1 for low crit and -1 for high crit
C_q	is quality pseudo cost (hr/m^3)
X_{ij}	is jth quality factor at ith shovel
C_p	is pseudo cost of low feed to plant (hr/m^3)
P_t	is target rate of plant feed
R_i	is digging rate at ith shovel
M_c	is 1st in/1st out average control mass, kg
SG	is specific gravity
T_c	is base control interval, hr
X_jL	is lower limit for quality factor j
X_jA	is running average value of quality factor j
X_jU	is upper limit for quality factor j

All pseudo costs are being chosen arbitrarily with respect to ($C_m < C_q < C_s < C_p$).

As the second segment of the LP model, DISPATCH tries to minimize total haulage capacity needed to meet shovel production coverage:

$$\min \mathrm{V} = \sum_{i=1}^{N_p} \left(P_i \times T_i \right) + \sum_{j=1}^{N_d} \left(P_j \times D_j \right) + N_e \times T_s \tag{5}$$

Subject to:

$$\sum_{k=1}^{N_{pi}} P_k = \sum_{k=1}^{N_{po}} P_{k'} \tag{6}$$

$$R_j = \sum_{k=1}^{N_{po}} P_{k'} \qquad \text{in the mine} \tag{7}$$

$$R_j \le \sum_{k=1}^{N_{po}} P_{k'} \qquad \text{at the stockpile} \tag{8}$$

$$P_j = Q_i \tag{9}$$

$$0 \le P_i \tag{10}$$

Where:

V	is total mine haulage (m^3)
N_p	is number of feasible haul routes
P_i	is haulage on path i which should be determined (m^3/hr)
T_i	is path i travel time (hr)
N_d	is number of dumps for mine haulage
P_j	is net haulage input to dump j (m^3/hr)
D_j	is the average dump time at dump j (hr)
N_e	is number of operating shovels
T_s	is fleet average truck size (m^3)
N_{pi}	is number of feasible input paths at node j
N_{po}	is number of feasible output paths at node j
P_k	is input path haulage (m^3/hr)
$P_{k'}$	is output path haulage (m^3/hr)
R_j	is limiting rate at node j (m^3/hr)

The model introduce first segment of the operation optimization as a pseudo cost based LP which is established on summation of costs in all four operational sector of the mine. The solution of the first segment present shovels production rates with respect to maximum digging rate of shovel (Eq. (2)), maximum capacity of the plant (Eq. (3)), and lower and upper bounds of the blending grade (Eq. (4)). The second segment LP maximizes production of the operation by allocating minimum number of trucks to each active route (Eq. (5)) to meet the routes productivity rate. Eq. (6) makes sure that input and output flow at each shovel and each dumping point are equal. Eqs.(7) and (8) guarantee material handling to meet grade requirement at plant cannot exceed shovels' digging rate working at stockpile. Coupling between two segments of the operation plan is attained by constraining total productivity of all routes servicing a shovel to be greater or equal to the shovel productivity (Eq.(9)). It should be mentioned here that both P and Q in Eq. (9) are vector. Finally, Eq. (10) ensures that all haul rate in the mine are nonnegative. The model follows current status of the mine. An advantage of the model is that the optimum production rate of each route is based on the volume of material not on number of trucks. That helps the dispatching step to send proper truck to cover the shortage. Major drawback of the model is that it does not consider stripping ratio (SR) limitation in the operation. However, most of the drawbacks of DISPATCH will arise in the real-time dispatching model which will be explained in more detail later on.

4.1.3. Real-time Dispatching

After solving upper stage (operation optimization) LP problem by implementing simplex method resulting optimum material flow rate on routes, White and Olson (1986) employs dynamic programming (DP) approach to send trucks to the proper destination. To do so, two list and three parameters are defined. List of needy shovels or LP-selected paths and list of trucks dumping material at discharge points or en route from a loading point to a destination are provided. Also, need-time (Eq.(11)) which is defined as the expected time for each path's next truck requirement formulated as follow:

$$need - time_i = L_j + F_{ij} \times \left(A_j - R_j \right) / P_i \tag{11}$$

Where:

L_j	is time last truck was allocated to the shovel j
F_{ij}	is flow rate of path i over the total flow rate into shovel j
A_j	is total haulage allocated by time L_j to shovel j
R_j	is haulage requirement of shovel j
P_i	is path flow rate (ton/hr or m^3/hr)

So, the neediest path which is on the top of the neediest shovels list will be the one with the shortest need-time. Then lost-ton is defined and formulated as a criterion to find the best truck for the neediest path from the truck list with Eq. (12):

$$lost - ton = \frac{truck\,size \times total\,rate}{required\,trucks} \times \left(truck\,idle + excess\,travel \right) + shovel\,rate \times shovel\,idle \tag{12}$$

Where:

Truck size is size of truck being assigned; Total rate is total digging rate of all shovels in mine; Required trucks is total required trucks in LP solution; Truck idle is expected truck idle time for this assignment; Excess travel is extra empty travel time to neediest shovel; Shovel rate is sum of all path rates into neediest shovel; Shovel idle is expected shovel idle time for this assignment.

Considering the lost-ton definition, best truck is the truck covering lost-ton of neediest shovel the most. After finding the best truck and assigning it to the neediest shovel, it is moved to the last position on the needy paths' list and the procedure is repeated for the second neediest which is now the neediest until all trucks on the list are assigned.

Defining a rolling time horizon when a sequence of assignment is needed is a benefit of the model. Because the information of the mine status which is being used in the model always is up to the minute. However, it does not consider effect of current truck assignment on the forthcoming truck matching, though all trucks previously sent to the shovels are considered. Another drawback of the model is that despite the authors claim, the solution method is not a DP. It is a heuristic rule solving each sub problem based on the best solution of previous sub problems and based on Alarie and

Gamache (2002), maybe it is because of the authors misunderstanding of Bellman's principal of optimality.

4.2. Academic Algorithms

Algorithms and models have been presented to solve open pit mine real-time operational plan in academic institutes are enclosed to the public. Herein, we reviewed the models those effects on the real-time open pit mines' operation optimization is undeniable.

4.2.1. Finding the Shortest Path

One of the first appearance of the operational problem in the literature of open pit mining is (Hauck, 1973), in which the shortest path was defined as the closet route from loading to discharge point time-wisely and based on the previous experience. In their non-linear model of solving upper stage problem as a network problem, Elbrond and Soumis (1987) and Soumis et al. (1989) solved a non-linear programming (NLP) network problem to find shortest path between all loading and discharge points. Among all, the most famous and interesting dispatching algorithm up until now which had been presented by Temeng et al. (1997) and Temeng et al. (1998) uses Dijkstra's algorithm of finding shortest path between source and sink to select the best route of connecting shovels to their destination.

4.2.2. Production Optimization and Truck Allocation

Most of the models presented in operational planning of open pit mines are focusing on upper stage or shovel and truck allocation part. The model developed by Soumis et al. (1989) performs the upper stage in two steps. As the first step, it fixes shovels' location by implementing combinatory mixed integer linear programming (MILP) model with respect to available trucks and the objective of maximizing the production and subject to quality constraints. By solving the MILP model it suggests some location for shovels to be seated on the computer screen, and it needs a human to make a decision on the shovels siting locations. Then after, as the second step of the algorithm Soumis et al. (1989) represents truck travel plan between shovels and dumping points by solving a NLP. The model's objective function consists of three major factors: 1- shovel production objective (computed shovel production); 2- available truck hours (computed truck hours) which includes truck waiting time as well; and 3- penalty for the deviation of the produced ore material from the blending objectives. Munirathinam and Yingling (1994) claim that there is an advantage of using NLP versus LP. The point is paths will not be on extreme. Because solution methods for solving LP models always look for the optimum solution on the corner of the feasible regions whereas NLP solution methods search for the optimum solution over the entire feasible region. As a result of implementing NLP model the flow rate will be split over paths which helps to achieve blending goals easier. Beside the advantage of the model, it is assumed that all trucks in the fleet are from the same capacity called homogenous truck fleet. However, generally truck fleet in mine is heterogeneous including different types and capacity of trucks. Second drawback of the model is assumption of fixed grade material in each mining faces. Whereas, stochastic nature of the ore material quality even in a single block is not ignorable (Osanloo et al., 2008).

However, the model was not presented clearly in the paper. Herein, other approaches of optimizing mine truck allocation for each of them there is at least one disclosed mathematical model are presented in following subsections: first queuing theory implementation is studied. As the second subsection algorithm based on transportation work is introduced. Then, models based on linear programming are overviewed before studying goal programming approach as a separated subsection. Finally, stochastic nature of the operation problem in open pit mine is considered in the last subsection.

4.2.2.1. Queuing Theory Approach

The first use of queuing theory in mining context is referred to (Koenigsberg, 1958) in which a room and pillar underground mine and a surface mine haulage system were modeled by using of queuing theory. The model presented by aforementioned author has a computational difficulties by the time fleet size increases. Afterwards, Barnes et al. (1978), Dallaire et al. (1978) and Carmichael (1987) applied queuing theory to solve truck – shovel problem in surface mines which has been followed by (Kappas and Yegulalp, 1991). Dallaire et al. (1978) defined mining operation as a system of several networks. After that, capacity of the transportation system and cycle time of each transportation unit (truck) is calculated by implementing mean value analysis method and based on recursive relations between waiting times. Major drawback of this model is production rate underestimation due to the approach under which it does not consider traveling time as infinite server queuing system. Second drawback of the model presented by Elbrond is that to implement the model in the operation a significant engineering judgment is needed. The model developed by Barnes et al. (1978) has the same drawback of Dallaire et al. (1978) model which does not consider traveling time as infinite server system, as well as disadvantage of using Erlang queuing model. Because Erlang distribution can approximate actual distribution with the coefficient variation of interval times less than one that can easily be violated in a real mining operation.

Kappas and Yegulalp (1991) offered a queuing theory model by considering truck – shovel system as a production network with regard to trucks as customer and shovels, crushers, waste dump, roads and maintenance service areas as servers. In their model, it is assumed that a mining system is a stochastic system with Markovian nature. Although it is stochastic, because of some parameters like service time distribution in different service areas, it is not Markovian (Newman et al., 2010).

Najor and Hagan (2006) applied queuing theory to analyze equipment (trucks and shovels) utilizations in the stochastic environment. Application of the model in an Australian case study shows that ignoring queue of trucks at hoppers (or plant capacity) causes overestimation of the production.

Later, Ercelebi and Bacetin (2009) represent a queuing theory model to allocate trucks in an open pit mine which can estimate some of mining systems performance parameters including number of trucks, throughput of the processing plant and waiting time. The model is presented below:

$$\left(\frac{N+M-1}{N}\right)=\frac{(N+M-1)!}{(M-1)N!} \tag{13}$$

$$P(n_1,n_2,\mathrm{k},n_M)=\frac{\mu_1^{N-n_1}}{\mu_2^{n_2}\mu_3^{n_3}\Lambda\mu_M^{n_M}}P(N,\mathrm{O},K,\mathrm{O})=\left(\frac{\mu_1}{\mu_1}\right)^{n_1}\left(\frac{\mu_1}{\mu_2}\right)^{n_2}\Lambda\left(\frac{\mu_1}{M_1}\right)^{n_M}P(N,\mathrm{O},K,\mathrm{O}) \tag{14}$$

$$\sum P(n_1,n_2,K,n_M)=1 \tag{15}$$

$$P(N,O,...,O)=\left[\sum\left(\frac{\mu_1}{\mu_1}\right)^{n_1}\left(\frac{\mu_1}{\mu_2}\right)^{n_2}\Lambda\left(\frac{\mu_1}{M_1}\right)^{n_M}\right]^{-1} \tag{16}$$

$$\sum_{i=1}^{M}n_i=N \tag{17}$$

$$\mathrm{Pr}[\mathit{phase\ i\ is\ working}]=\eta_i=1-\sum P(n_1,n_2,K,n_{i-1},\mathrm{O},\mathrm{n}_{i+1},K,n_M) \tag{18}$$

$$L_{qi}=\sum n_iP(n_1,n_2,K,n_M)-\sum P(n_1,n_2,K,n_M) \tag{19}$$

$$W_i=W_{qi}+\frac{1}{\mu_i} \tag{20}$$

$$LCT = \sum_{i=1}^{M} (W_{qi} + \frac{1}{\mu_i}) \tag{21}$$

$$\text{Production} = \frac{\textit{time period of } \text{interest}}{\textit{average cycle time}} \times N \times \textit{truck capacity} \tag{22}$$

$$\text{Production} = \text{time period of interest} \times \eta_{shovel} \times \mu_{shovel} \times \textit{truck capacity} \tag{23}$$

$$C = \frac{C_1 + C_2 N}{\textit{unit production} \times \textit{truck capacity}} \tag{24}$$

Where:

N	is total number of trucks
M	is total number of service centers (herein: loaders, loaded haul roads, empty haul roads, dump sites)
n_i	is the number of trucks in ith service center
P	is the steady state probability (Eq.(16))
μ_i	is service rate at ith service center
η_i	computes the probability that service center ith is working (utilization) (Eq.(18))
L_{qi}	calculates the expected number of trucks in the queue at the ith service center (Eq.(19))
W_{qi}	is the expected time a truck spends at service center (= L_{qi}/ η_i μ_i)
W_i	estimates the expected time that a truck spends in the ith service center Eq.(20)
LCT	is the average total cycle time for a truck to complete M service centers (Eq.(21))
C_1	is the cost per unit of shovel (including capital and operating costs)
C_2	is the cost per unit time of truck (including capital and operating costs)
C	is total cost for unit production

Average cycle time is sum of load time, dump time, queuing time at the shovel, queuing time at the dump, loaded haul time, and empty haul time. Eqs.(13), (14), (15), (16), and (17) show the procedure from which probability of each phase utilization is being account. Eq.(22) or (23) are implemented to find production per unit of time and Eq.(24) computes total cost per tonne of material extracted.

Their model has some disadvantages such as: they assumed all stochastic procedures in the operation are Markovian which is not true, the fleet is consisting of the same size (homogenous fleet), and truck cycle time is calculated based on locked-in allocation which does not care about the time a truck needs to reach the route.

To sum up, although queuing theory is a powerful approach, but by advances in the simulation, researchers prefer to use simulation as a tool to cover stochastic nature of the problems instead of queuing theory in the field of mine operation optimization.

4.2.2.2. Transportation Approach

Li (1990) says that an optimum material flow on a path should minimize total transportation work (Eq.(25)) with respect to Eqs.(26) and (27) those ensure the model will meet stripping ratio, Eq.(28) to meet grade requirement and Eq.(29) to ensure that number of trucks input in a loading or discharge point is equal to number of trucks come out of that point. Transportation work is defined as the distance material is transported multiply by the amount of the material. The transportation model was presented by Li (1990) for five shovels is as follow:

$$\min W = \sum_{i\in S_1}\sum_{j\in S_2\cup S_3} X_{ij}(Z_1+Z_2)\sum_{k=1}^{K_{ij}} f_{ij}^{(k)}D_{ij}^{(k)} + \sum_{i\in S_4}\sum_{j\in S_5} X_{ij}(Z_1+Z_3)\sum_{k=1}^{K_{ij}} f_{ij}^{(k)}D_{ij}^{(k)} + \sum_{i\in S_2\cup S_3\cup S_5}\sum_{j\in S_1\cup S_4} X_{ij}Z_1\sum_{k=1}^{K_{ij}} f_{ij}^{(k)}D_{ij}^{(k)} \tag{25}$$

Subject to:

$$P_i / T \le \sum_{j\in S_2\cup S_3} X_{ij}Z_2 \quad for\, i\in S_1 \tag{26}$$

$$P_i / T \le \sum_{j\in S_5} X_{ij}Z_3 \quad for\ i\in S_4 \tag{27}$$

$$\sum_{i\in S_1}\alpha_i^{(q)}\sum_{j\in S_2} X_{ij} = \alpha^{(q)}\sum_{i\in S_1}\sum_{j\in S_2} X_{ij} \qquad for\ q=1,2,...,Q \tag{28}$$

$$\sum_{i\in S_{.j}} X_{ij} = \sum_{k\in S_{j.}} X_{jk} \qquad for\ j\in\bigcup_{i=1}^{5} S_i \tag{29}$$

Where:

S_1	is set of ore shovels
S_2	is set of ore discharge points
S_3	is set of stockpile points
S_4	is set of waste shovels
S_5	is set of waste disposing points
X_{ij}	is the truck flow over path from ith loading point to jth discharge point
K_{ij}	is total number of segments on path ij
$D_{ij}^{(k)}$	is the length of kth segment on ijth route
$f_{ij}^{(k)}$	is the road resistance factor of kth segment of ijth path
Z_1	is net truck weight
Z_2	is ore payload
Z_3	is waste payload
T	is planning period over which number of loading and dumping points do not change
P_i	is amount of material to be transported from ith loading point in T time
Q	is total number of ore quality indicator
$\alpha_i^{(q)}$	is ore quality of indicator q at ith loading point
$\alpha^{(q)}$	is required ore quality of indicator q at processing plant
$S_{.j}$	is set of all loading and discharging points which have path to jth discharge point

$S_{j.}$ is set of all loading and discharge points constitute feasible paths from j

The method implements aforementioned LP model to allocate optimal number of trucks to a route meeting its productivity rate. The model presented is based on five shovel fleet but author claims that the model can be implemented in a mine with higher number of loading point as well. It consider productivity of each shovel and also blending requirement. One major drawback of the model is that total model of operational plan including upper and lower stages are based on homogenous fleet. However, it will not guarantee optimality in real projects where the fleet is heterogeneous because it allocate trucks to each shovel based on assumption of the same capacity whereas they are not. Another major drawback is that the model does not consider truck breakdown as a major event that changes mine status. However, truck breakdown will not allow the operation to achieve the maximum production planned.

4.2.2.3. Linear Programming Approach

LP and specially MILP has been implemented in the open pit mine operation optimization more than any other approaches. The general LP model implemented in mine operation optimization was developed by Bonates (1992) who introduced an LP model to maximize shovel productivity (Eq.(30)) as follow:

$$\max Z = \sum_{i=1}^{n} P_i X_i + \sum_{j=1}^{m} Q_j X_j \tag{30}$$

Subject to:

$$\sum_{i=1}^{n} X_i \le CC \tag{31}$$

$$\sum_{i=1}^{n} \left[G_u - G_i \right] X_i \ge 0 \tag{32}$$

$$\sum_{i=1}^{n} \left[G_i - G_l \right] X_i \ge 0 \tag{33}$$

$$X_k < MAXP_k \quad for \;\; k = 1, 2, ..., n+m \tag{34}$$

$$X_k > MINP_k \quad for \;\; k = 1, 2, ..., n+m \tag{35}$$

$$\sum_{k=1}^{n+m} \left[X_k / B_k \right] \le TT \tag{36}$$

$$R_u \sum_{i=1}^{n} X_i - \sum_{j=1}^{m} X_j \ge 0 \tag{37}$$

$$R_l \sum_{i=1}^{n} X_i - \sum_{j=1}^{m} X_j \le 0 \tag{38}$$

Where:

i is index of shovels in ore

j is index of shovels in waste

n is total number of shovels in ore

m is total number of shovels in waste

k	is general shovel index
CC	is crusher capacity
X_i	is ore production per period of ith shovel
X_j	is waste production per period of jth shovel
P_i	is priority of ith shovel for production
Q_j	is priority of jth shovel for production
G_u	is material quality upper limit
G_l	is material quality lower limit
G_i	is material grade at ith shovel
$MAXP_k$	is maximum digging rate at kth shovel
$MINP_k$	is minimum production rate at kth shovel
B_k	is linear approximation for trucks working with kth shovel between $MINP_k$ and $MAXP_k$
TT	is total number of available trucks over the time horizon
R_l	is lower limit of SR
R_u	is upper limit of SR

Constraint (31) makes sure that total production of shovels working in ore area do not exceed maximum capacity of crusher. Eq.(32) and (33) guarantee that ore quality is within the prescribed limits. Constraints (34) and (35) ensure total production of each shovel over the time period will not deviate from minimum and maximum digging rate of the shovel. Eq.(36) ensures total number of truck is being used over the time horizon do not proceed total number of available trucks. Constraints (37) and (38) ensure stripping ratio requirement will be met.

The LP model was presented to be employed in small to medium size mines. The objective is to maximize the production of all shovels. The model consider required grade interval for feeding the plant. It also account for stripping ratio and relative priority of shovels specially ones working on ore faces. Nevertheless, it was assumed that shovels production will increase linearly by increasing the number of trucks. However, in heterogeneous fleet by adding trucks with different sizes to the available fleet production rate will increase nonlinearly up to its maximum production rate. Another major drawback of the model is that it is necessary to add stockpiling (re-handling) to the objective as well.

Gurgur et al (2011) proposed an LP model of operation optimization that helps to minimize deviation of the operation from the strategically set targets in short- and long-term schedules. To link operation plan to the strategic ones the model provides shovel assignment. Advantage of model is that it account for available trucks of fleet in each time period. Second advantage of the model is that it is a lifelong model which considers the mine as a multi period task. As a result, effects of current operations on the next ones are taken into account. There is a major disadvantage of the model presented by (Gurgur et al., 2011) that pushs it away from optimality. Costs and lost tons associated with the shovel movement during the operation is not considered. Another drawback of the model is using continuous variables in the discrete production operation, which provides the rates of material transported using various trucks. The only constraint relating the flow rate with the capacities of the trucks is the available fleet constraint, which though limits the total production transported by trucks with the maximum transportation possible, cannot provide exact measure of the number of truck trips required.

Ta et al. (2013) developed a mixed integer linear programming (MILP) model to allocate trucks of a fleet to different shovels based on probability of shovels' idle time. The probability of the ide time approximated by defining shovel as a server of the mine as a G/G/1 /y finite-source system. The objective of the model is to minimize total number of trucks. The model was implemented in a simulation mode of an oil sand mine. Results of the simulations show that in some cases idle probability of some shovels goes up to 40% to 60% illustrating that the model does not provide a reliable open-pit mine equipment allocation. Second drawback of the model is: Although it is claimed by authors that the model has the ability of being used in heterogeneous fleet, regarding to the simulation results, it does not offer a realistic combination of the trucks with different sizes available at the fleet.

Mena et al. (2013) defined a knapsack problem which tries to maximize cumulative truck fleet production by a fixed time horizon. Their main aim was to allocate available trucks to the route requesting for a truck. To do so, they used equipment availability function as a part of objective function coefficient. They multiplied productivity of truck on each route by the availability of the truck and tried to maximize the problem in this way. Then by implementing simulation procedure they solve the model for a period horizon of one week. Then compared results of their model with the result of general model without considering availability. The comparison showed decrease in the productivity of the fleet because the enhanced model was more accurate. Advantage of the model is that it uses each truck with its own availability and in this model there is no equipment with 100% availability. Major drawback of the model was that at the time a certain number of trucks fail or go out of performance for maintenance repair, then the system becomes infeasible and the optimizer is not able to find an optimal solution problem. Another disadvantage of the model is that only availability of the trucks is inputted in the optimization problem. However, priority in mining system is with bigger equipment and it is needed to add availability of all equipment who plays a role in the production procedure. Along with above cons, the blending requirement of the plant feed is not considered in the model as well.

The most resent model based on the LP has been presented by Chang et al. (2015). The model schedules trucks over a shift by implementing MILP with the objective of maximizing transportation revenue. Then a heuristic rule is implemented to solve the model. They also take into account transport priority. The model is based on homogenous truck fleet which is far from reality and cause non-optimality of the model results on a real system. The model does not consider stripping ratio requirement as well as ignoring stochastic nature of grade distribution. Plant capacity and feed head grade are ignored as well.

One of the major drawbacks of all models based on linear programming is that: to consider limitations of operation such as stripping ratio and required feed grade the models have to define an acceptable range. However, it pushes the operation far behind optimality especially if plant feed grade requirement changes.

4.2.2.4. Goal Programming Approach

The Goal Programming (GP) first introduced by Charnes and Cooper (1955) and (1961). In the simplest version of GP, the designer prepares some goals he or she wishes to achieve for each objective function. Then, the optimum solution is the set which minimize deviations from the goals has been set which means that it does not maximize or minimize an specific objective, it tries to find an specific goal value of those objectives, though (Caramia and Dell'Olmo, 2008). The general GP model for multi-objective problems (Eq.(39)) is as follow (Rao, 2009):

$$\min\left[\sum_{j=1}^{k}(d_j^{+}+d_j^{-})^{p}\right]^{1/p} \quad , \; p\geq 1 \tag{39}$$

Subject to:

$$g_j(X) \le 0 \qquad for\ j = 1,2,\ldots,m \tag{40}$$

$$f_j(X) + d_j^+ - d_j^- = b_j \quad for\ \ j = 1,2,\ldots,k \tag{41}$$

$$d_j^+ \ge 0 \qquad for\ \ j = 1,2,\ldots,k \tag{42}$$

$$d_j^- \ge 0 \qquad for\ \ j = 1,2,\ldots,k \tag{43}$$

$$d_j^+ d_j^- = 0 \qquad for\ \ j = 1,2,\ldots,\mathrm{k} \tag{44}$$

Where:

b_j	is the set of goals
$d_j^+\ and\ d_j^-$	are the underachievement and over achievement of the jth goal
p	is the value chosen by designer based on utility function

Eq. (40) shows general format of constraints. By algebraic summation of optimum results and deviations the goals will be achieved (Eq.(41)). Eq.(42), (43) and (44) are defining negative and positive deviation from each goal.

In the mining operation optimization there exist variety of goals to be achieved such as production maximization and maintenance of ore quality between the desired limits (Temeng et al., 1998), optimization of the processing plant utilization and minimization of trucks and shovels movement costs (Upadhyay and Askari-Nasab, 2015).

Temeng et al. (1998) formulated a model of open pit mine operation optimization based on GP which is presented below:

$$\min P_1 \sum_{i=1}^{n_s} d_i^- + P_2 \sum_{k=1}^{n_q} \sum_{j=1}^{n_c} (c_{ij}^+ + c_{kj}^-) \tag{45}$$

Subject to:

$$\sum_{j=1}^{n_d} x_{ij} + d_i^- = M_i \qquad for \quad i = 1,\ldots,n_s \tag{46}$$

$$\sum_{j=1}^{n_d} x_{ij} \ge B_i \qquad for \quad i = 1,\ldots,n_s \tag{47}$$

$$\sum_{i=1}^{n_s} x_{ij} \le C_j \qquad for \quad i = 1,\ldots,n_d \tag{48}$$

$$\sum_{j=1}^{n_d} y_{ji} = \sum_{j=1}^{n_d} x_{ij} \qquad for \quad i = 1,\ldots,n_s \tag{49}$$

$$\sum_{i=1}^{n_s} x_{ij} = \sum_{i=1}^{n_s} y_{ji} \qquad for \quad i = 1,\ldots,n_d \tag{50}$$

$$\sum_{i=1}^{n_{os}} G_{ik} x_{ij} + c_{kj}^- - c_{kj}^+ = Q_{kj} \sum_{i=1}^{n_{os}} x_{ij} \qquad for \quad k = 1,\ldots,n_q \quad j = 1,\ldots,n_c \tag{51}$$

$$c_{kj}^{-} \le (Q_{kj} - L_{kj})\sum_{i=1}^{n_{os}} x_{ij} \qquad for \quad k = 1,\dots,n_q \quad j = 1,\dots,n_c \tag{52}$$

$$c_{kj}^{+} \le (Q_{kj} - U_{kj})\sum_{i=1}^{n_{os}} x_{ij} \qquad for \quad k = 1,\dots,n_q \quad j = 1,\dots,n_c \tag{53}$$

$$R_L \le \frac{\sum_{i=n_{os}+1}^{n_s} \sum_{j=n_c+1}^{n_d} x_{ij}}{\sum_{i=1}^{n_{os}} \sum_{j=1}^{n_c} x_{ij}} \le R_U \tag{54}$$

$$\sum_{i=1}^{n_s}\sum_{j=1}^{n_d} H_{ij} x_{ij} + \sum_{i=1}^{n_s}\sum_{j=1}^{n_d} D_j x_{ij} + \sum_{j=1}^{n_d}\sum_{i=1}^{n_s} R_{ji} y_{ji} + \sum_{j=1}^{n_d}\sum_{i=1}^{n_s} S_i y_{ji} \le N.T \tag{55}$$

$$d_i^{-}, x_{ij}, y_{ij}, c_{kj}^{+}, c_{kj}^{-} \ge 0 \tag{56}$$

Where:

P_1	is priority factor for production
P_2	is priority factor for grade control
d_i^{-}	is ith shovel production negative deviation variable
c_{kj}^{+} *and* c_{kj}^{-}	are positive and negative deviation from ore grade indicator k at jth crusher
n_s	is number of shovels
n_q	is number of quality identifiers
n_c	is number of the crushers
n_d	is total number of destinations
n_{os}	is number of shovels working at ore faces
x_{ij}	is the production to be assigned to the ijth path connecting ith shovel to jth discharge point in each shift
y_{ij}	is capacity of truck which is to be assigned from jth dumping point to ith shovel per shift
M_i	is the maximum production of ith shovel per shift
B_i	is the minimum production of ith shovel per shift
C_j	is the maximum available capacity of jth discharge point per shift
G_{ik}	is the average ore quality indicator k at ith shovel
Q_{kj}	is the target ore quality indicator k at jth crusher
L_{kj}	is the prescribed lower limit of ore quality indicator k at jth crusher
U_{kj}	is the prescribed upper limit of ore quality indicator k at jth crusher
R_L and R_U	are prescribed lower and upper bounds of required stripping ratio

H_{ij}	is the average travel time from ith shovel to jth discharge point
D_j	is the average dumping time at jth destination including spot time
R_{ji}	is the average travel time from jth discharge point to ith shovel
S_i	is the average loading time at ith shovel including spot time
N	is number of trucks
T	is weighted average truck payload

The model maximize shovel production and ensure ore grade requirement achieved as much as possible (Eq.(45)). Eqs.(46) and (47) ensures that total material transported from ith shovel cannot exceed shovel's digging rate and will not be less than its minimum digging rate. Eq.(48) makes sure that total material dumped in each discharge point cannot surpass its maximum capacity. Eqs.(49) and (50) ensure that number of trucks travels into a point is equal to number of trucks come out of the point. Eqs. (51), (52) and (53) guarantee ore quality requirement at plant. Eq.(54) conserves the production between required stripping ratio. Eq.(55) ensures that total production cannot exceed total truck capacity available. The main advantage of GP model developed by (Temeng et al., 1998) is that it optimizes two major goals of the open pit operation simultaneously without neglecting any of them. Besides covering the objective function drawbacks of previous models it covers another disadvantage of LP models which is defining upper and lower limits for the target grade of material are being sent to the plant. As it was introduced before, in LP models it is usual to control the grade by imposing it between upper and lower limit. Let us assume that objective is to maximize the production. Then truck assignment to the shovel closer to the crusher which results shorter truck cycle time will be higher. If the average grade at these closer faces are pretty close to one of the allowed grade boundaries, then whatever the dispatching algorithm is controlling the feed grade within the interval is difficult. As a result, existing of stockpile and subsequently re-handling cost associated with it is undeniable. However, the model has some disadvantages. It does not consider all the goals are supposed to be met in an open pit mine operation such as equipment movement costs, of which some of them are covered by Upadhyay and Askari-Nasab (2015). The model the mining operation as a multi-period operation which needs to meet strategic goals of the project. It does not consider stochastic nature of the grade of material are feeding to the plant as well. The most resent open pit operation optimization model based on GP can be found in (Upadhyay and Askari-Nasab, 2015) where the authors enhanced aforementioned model's objective with adding two new goals. The newly added goals are minimizing the deviation of calculated plant feed to desire feed and minimizing cost of both trucks and shovels operation, respectively.

4.2.2.5. Stochastic Approach

Ta et al. (2005) implemented a chance-constrained stochastic optimization to allocate trucks in an open pit mines as a part of upper stage in mine operational plan. They also used an updater to renew the model and parameters by the time shift or status of the mine changes. The presented model considers truck load and its cycle time as stochastic parameters. The decision variables in the model are number and types of trucks allocated to the shovels. Authors claim that stochastic model they presented can be solved by converting it to a quadratic deterministic model and implementation of mixed integer nonlinear programming techniques and solvers but it is time consuming. So the initial model was divided into two sub model. The sub models were solved to allocate discrete number of trucks to each loader. The main model is as follow:

$$\text{Minimize Truck Resource} = \sum_{s}\sum_{d}\sum_{g} K(g)X(s,d,g) \qquad (truck\,units) \tag{57}$$

Subject to:

$$\text{Prob}\left\{V_o + H\left[V_{Truck} - V_{Extraction}\right] \geq V_{Min}\right\} \geq \alpha \tag{58}$$

$$V_{Truck} = \sum_s \sum_d \sum_g \frac{60}{\tau_o(s,d,g)} L_o(\text{s,d,g})\,\text{X}(\text{s,d,g}) \qquad \text{(tonnes/ hr)} \tag{59}$$

$$\sum_d \sum_g \frac{60}{\overline{\tau}_o(s,d,g)} \overline{L}_o(\text{s,d,g})\,\text{X}(\text{s,d,g}) \leq C_{Shovel}(s) \qquad \text{(tonnes/ hr)} \tag{60}$$

$$\sum_s \sum_d \text{X}(\text{s,d,g}) \leq R(g) \tag{61}$$

$$X(s,d,g) \geq 0 \tag{62}$$

Eq.(58) ensures confidence level of the model is more than or equal to predefined level (α). Eq.(59) calculates total volume a truck can transport in a unit of time (hr). Eq.(60) aims to limit trucks at shovel based on the shovel capacity. Eqs.(61) and (62) limit number of trucks in use to the available trucks in the fleet. The first sub model which is a probabilistic chance-constrained model is as follow:

$$\text{Minimize Truck Resource}(1) = \sum_s \sum_d \sum_g K(g)X(s,d,g) \tag{63}$$

Subject to:

$$V_{Truck} = \sum_d \sum_g \frac{60}{\overline{\tau}_o(s,d,g)} \overline{L}_o(\text{s,d,g})\,\text{X}(\text{s,d,g}) \tag{64}$$

$$\text{Prob}\left\{V_o + H\left[V_{Truck} - V_{Extraction}\right] \geq V_{Min}\right\} \geq \alpha \tag{65}$$

$$V_{Truck} \leq C_{Shovel}(s) \tag{66}$$

$$V_{Truck} \geq mC_{Shovel}(s) \tag{67}$$

$$\sum_s \sum_d \text{X}(\text{s,d,g}) \leq R(g) \tag{68}$$

$$X(s,d,g) \geq 0 \tag{69}$$

First sub problem is almost the same as the general problem. Except for constraint (67) which maintains the solution from assignment of zero truck to shovels is the only difference. Also minimum ore throughput from the shovels is maintained. The model must be simplified as a nonlinear deterministic model and be solved by using of nonlinear techniques. The model provides a continuous amount for number of trucks which must be a discrete number. To do so, second sub problem as follow was presented:

$$\text{Minimize Truck Resource}(2) = \sum_s \sum_d \sum_g K(g)Y(s,d,g) \tag{70}$$

Subject to:

$$\sum_s \sum_d \sum_g K(g)Y(s,d,g) \geq \text{Truck Resource}(1) \tag{71}$$

$$\sum_d \sum_g \frac{60}{\overline{\tau}_o(s,d,g)} \overline{L}_o(\text{s,d,g})\,\text{Y}(\text{s,d,g}) \leq C_{Shovel}(s) \tag{72}$$

$$\sum_{d}\sum_{g}\frac{60}{\overline{\tau}_o(s,d,g)}\overline{L}_o(\mathrm{s,d,g})\,\mathrm{Y(s,d,g)} \geq mC_{Shovel}(s) \tag{73}$$

$$\sum_{s}\sum_{d}\mathrm{Y(s,d,g)} \leq R(g) \tag{74}$$

$$\begin{aligned} &i=1: Y^{(i)}(x,d,g) \geq 0 \\ &i=2,3,...: Y^{(i-1)}(x,d,g)-1 \leq Y^{(i)}(x,d,g) \leq Y^{(i-1)}(x,d,g)+1 \end{aligned} \tag{75}$$

$$Y(s,d,g) \geq 0 \tag{76}$$

Where:

s	is shovel type; d is type of discharge point
g	is truck type
K(g)	is cost coefficient of truck type g (for the truck type g with the smallest capacity K(g)=1 and for the rest it is calculated based on the smallest truck capacity. For example, in a fleet consisting of 240 ton and 320 ton capacity trucks K(240)=1 and K(320)=1.33)
X(s,d,g)	is number of truck type g assigned to shovel s and dump d (fractional or theoretical)
Y(s,d,g)	is number of truck type g assigned to shovel s and dump d (discrete)
L_o(s,d,g)	is the truck type g capacity working on route connecting shovel s to dump d
$\tau_o(s,d,g)$	is ore truck cycle time (minute)
V_o	is initial surge volume
V_{Truck} & $V_{Extraction}$	are ore production rate that goes in and out of surge per hour
C_{Shovel}(s)	is capacity of shovel s (tonnes/hr)
D_w	is amount of waste needs to be handled per hour
R(g)	is tha available number of type g truck
H	is number of hours in each period of concern
m	is used to specify the minimum amount of ore to be mined by the working shovels $(0 \leq m \leq 1 \quad ton/hr)$

Constraint (71) defines the lower bound of the objective function. Eqs.(72), (73) and (74) are the same as (66), (67) and (68) in the first sub model with the exception of number of trucks being discrete. Eq.(75) helps to move to the next time period realistically.

The objective function value of the first sub problem helps to define a lower limit for the objective function value of the second sub problem. To move to the next time horizon, constraint (75) is defined to ensure gradual transition of allocation from the current period of time. Although the model provides a good conceptual background for stochastic optimization approach to solve the multi-stage optimization problem, it takes into account the probabilistic nature of truck travel times only. Also the model formulation is very much specific to a mining case and cannot be generalized to other mining systems.

4.2.3. Real-time Dispatching

Real-time decision making on the destination of trucks in a mining operation was first used in early 1960s with implementation of radio communication tools to link between dispatcher and trucks operators in a fixed truck allocation mine. However, based on utilization of computer real-time fleet management in mining operation systems are divided into three major categories: locked-in or fixed allocation, semi-automated and fully automated systems. In the locked-in method there is no effort for dispatching the transportation units. Semi-automated dispatching which has been developing by increasing the computer usage in mining sector is divided into two different classes: passive and active. In the earlier class computer just displays current mine operation information and does not have any role on decision making procedure. However, in the later class computers use current mine status information as inputs and process them based on predefined models and suggest list of assignments to the dispatcher and leave the decision to be make for humans. In the automated dispatching data of the current mine status and condition and position of the equipment within the operation are collected into a main computer server and it sends the assignment to trucks after solving some heuristics or mathematical programs. What we review here is the last class where computers receive data, process them and assign the trucks to the next destinations.

There are two major approaches governing dispatching procedure: Assignment problem approach and transportation problem approach from those the first one is a subcategory of the transportation problem in the operations research context.

4.2.3.1. Algorithms based on Assignment Problem

A general assignment problem is a balanced transportation problem in which all demands and sources have capacity of one unit. In each assignment problem there is a cost matrix that consists of the costs associated with assigning each supply to each demand. The objective of each assignment model is to minimize cost of allocating supplies to demands. In mining context assignment problem has been used mostly to dispatch trucks as supply to shovels or dumping points as demand. The objective in mining truck dispatching based on assignment model is to minimize shovel idle time, truck waiting time, inter-truck time, and so on. In comparison with the other approach, almost all real-time truck dispatching models in both industrial and academic research area are based on assignment problem.

Hauck (1973) implemented a sequence of assignment problem to dispatch the trucks need destination. The objective function of his model is to minimize total idle time of shovels which minimize lost ton of the operation subsequently. The sub problem that is solved in each assignment request is as follow:

$$\min \sum_i \sum_j W_{ij}(t_k) X_{ij}(t_k) \tag{77}$$

Subject to:

$$\sum_j X_{ij}(t_k) \le 1 \qquad for \quad i = 1,...,m \tag{78}$$

$$\sum_i X_{ij}(t_k) = 1 \qquad for \quad j = 1,...,n \tag{79}$$

$$X_{ij}(t_k) \in D_k \tag{80}$$

Where:

$$X_{ij}(t_k) = \begin{cases} 1 & if\ truck\ i\ is\ loaded\ by\ shovel\ j\ and\ departed\ at\ time\ t_k \\ 0 & otherwise \end{cases}$$

$W_{ij}(t_k)$ is lost ton due to idle time caused by assigning ith truck to jth shovel at time t_k; D_k is representative of a situation will be explained later on.

The model tries to minimize lost ton due to idle periods. Constraint (78) guarantees that each truck is assigned to at most one shovel and constraint (79) ensures that each shovel is assigned exactly one truck. Eq.(80) ensures that a truck to be assigned meets all requirements

Two main disadvantages of the dispatching part of Hauck's model are: firstly, assignment is not as accurate as possible because the decisions are made now will not be recomputed unless number of available trucks change. As a result, the assignment decision is not up to the minute. Secondly, the model is a sub model of a large model which uses result of last stage of the total model above dispatching. The last stage above dispatching decision making model itself is an optimum result of its previous sub model. So, the dispatching model is not able to use DP to solve assignment problem because it does not have possibility of using all possible solutions of previous stages and only uses the optimum solution of those stages.

Soumis et al. (1989) developed an assignment model that consider 10-15 forthcoming trucks and their effects on current assignment. The objective of the model is to minimize sum of squared deviation of estimated waiting time of trucks from the planned waiting time. The model finds 10-15 next trucks based on average travel time, discharge time, and loading time and shovel inter-truck waiting time. After assignment of current truck, all 10-15 trucks which had been used for the assignment are erased. The procedure will repeat when next assignment is requested. The main advantage of the method is that it considers effects of forthcoming trucks on the current assignment. However, assumption of homogeneous fleet is a drawback of the model. Assuming homogenous fleet of trucks in a multistage model of truck dispatching cause a considerable deviation from the reality. The reason behind such a deviation is to use homogenous fleet in the lower stage (real-time dispatching level) it is necessary to model upper stage (operation optimization level) considering homogenous truck fleet as well. Consequently, optimized production rate resulted from upper stage is far from the one in reality because in reality trucks in the fleet are from different size in most of the fleets (Alarie and Gamache, 2002). But, based on Lizotte et al. (1987) to implement a multistage dispatching algorithm for an open pit mine operation the production plan should represents the mine as close to reality as possible to have an optimal plan. The second major drawback of the model which happens in almost all of the dispatching models based on assignment problem is that, although they account for upcoming trucks for current assignment request, effects of current assignment on forthcoming trucks are not accounted for.

Ercelebi and Bascetin (2009) after providing optimum truck allocation by using of queuing theory implemented assignment problem approach based on the model was presented by White and Olson (1986) to dispatch trucks requesting a new destination. Lizotte et al. (1987) in their semi-automated model first provided a simulation model of the case study where by the time a truck needs assignment, three dispatching heuristic based on assignment problem are solved and the results of the simulation are presented on the board in a table beside the result of fixed allocation method and leave the decision for the dispatcher.

All dispatching heuristic rules in the literature that are grounded on maximize truck utilization in which a truck is sent to the shovel where it is supposed to be loaded first follow assignment problem. Although such an objective improves production in comparison with locked-in non-dispatching operation, they have some drawback including: ore quality and stripping ratio are not taken into account. Another major drawback of these types of algorithms is that it tries to send trucks to the shorter routes and as a result shovels sitting on further mining faces will idle more (Tu and Hucka, 1985; Lizotte et al., 1987). All dispatching rules in the literature based on maximum utilization of the shovels in which truck is sent to the shovel that is supposed to idle longer by the time truck reaches the face are following assignment problem as well.

To sum up, although implementing assignment problem provides fast solution to for real-time truck dispatching in mining operations, it has two major drawbacks arising from the nature of the assignment problem: The main drawbacks of algorithms based on assignment problem is that at each time just one truck is assigned to each shovel even if a shovel is far behind its production target and needs more than one truck. As the second drawback it can be say that despite claims of some authors, it is not able to consider effects of forthcoming trucks.

4.2.3.2. Algorithms based on Transportation Problem

A transportation problem in the optimization context is described as follow (Winston, 2003):

1- A set of supply points (m);

2- A set of demand points (n);

3- Cost associated with transporting material from the supply point i to the demand point j.

Let x_{ij} is number of units shipped from the supply point i to the demand point j, then general formulation of the transportation problem is:

$$\max \text{or} \min \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij} \tag{81}$$

Subject to:

$$\sum_{j=1}^{m} x_{ij} \le s_i \qquad for \quad i=1,\dots,n \qquad \text{Supply constraints} \tag{82}$$

$$\sum_{i=1}^{n} x_{ij} \ge d_j \qquad for \quad j=1,\dots,m \qquad \text{Demand constraints} \tag{83}$$

$$x_{ij} \ge 0 \qquad for \quad \begin{array}{l} i=1,\dots,n \\ j=1,\dots,m \end{array} \tag{84}$$

To have a feasible solution, each transportation model must be constrained as:

$$\sum_{i=1}^{m} s_i \ge \sum_{j=1}^{n} d_j \tag{85}$$

The model tries to minimize total costs of the decision to be made (Eq.(81)). Constraint (82) makes sure that total material sent to different sink points cannot exceed ith source capacity. Constraint (83) ensure that jth sink will meet its demand. Constraint (84) limits the material to be handled to non-negativity. The most reliable algorithm of the real-time truck dispatching in open pit mine is the model was developed based on transportation problem by Temeng et al. (1997). The procedure of truck dispatching by using of Temeng et al. (1997) transportation algorithm is as follow:

Firstly a needy shovel is defined as a shovel uses a route that up until now has a cumulative production behind its production target. Or in the other word, a non-needy shovel is a shovel that cumulative production of all routes ending to it are above or equal to the target.

To find the needy shovels we first calculate current mean of tonnage ratio by using of Eq.(86):

$$R = \frac{1}{nm}\sum_{i=1}^{n}\sum_{j=1}^{m} R_{ij} \tag{86}$$

Where:

$R_{ij} = x_{ij} / T_{ij}$; x_{ij} is current cumulative tonnage on path ij; T_{ij} tonnage be assigned to the path ij which links ith shovel to jth dump.

Set current mean as the target ratio of each route. Then for each route define d_{ij} (Eq.(87)) as deviation of the route ij from the target production:

$$d_{ij} = R_{ij} - R \tag{87}$$

Now, a needy shovel is a shovel with $d_{ij} < 0$ (negative deviation).

Secondly, number of trucks each needy shovel requires is determined. To do so, at first Eq.(88) or (89) are being implemented to calculate y_{ij} as the tonnage behind the target of the route ij:

$$\frac{x_{ij} + y_{ij}}{T_{ij}} = R \tag{88}$$

$$y_{ij} = RT_{ij} - x_{ij} \tag{89}$$

Then, a basic truck capacity (small, large, or an average of them) is chosen based on some statistical analysis. Before, the demand of each route is found by using of Eq.(90):

$$M_{ij} = \left\lceil \frac{y_{ij}}{C_1} \right\rceil \tag{90}$$

Where:

M_{ij} is the demand of each route ij; C_1 is the larger truck capacity in the fleet consisting of two different truck sizes; $\lceil x \rceil$ is the smallest integer $\geq$ x.

Finally, the demand for each shovel will be Eq.(91):

$$D_i = \sum_{j=1}^{m} M_{ij} \qquad i = 1, \dots, n \tag{91}$$

And if the demand of ith shovel is D_i, then Eq. (92) is being used to calculate total demand of the operation at current status:

$$D = \sum_{i=1}^{n} D_i \tag{92}$$

In which D must be less than or equal to number of trucks available for the assignment and if it is not, a cut-off value for required tonnage should be used that selects those shovels as needy ones with relatively higher negative tonnage.

Finally Eq.(93) presents the model to assign trucks that tries to minimize total cumulative waiting time associated with the assignment:

$$\min \sum_{k=1}^{l} \sum_{i=1}^{n} W_{ik} X_{ik} \tag{93}$$

Subject to:

$$\sum_{i=1}^{n} X_{ik} \leq S_k \qquad for \quad k = 1, \dots, l \tag{94}$$

$$\sum_{k=1}^{l} X_{ik} \geq D_i \qquad for \quad i = 1,...,n \tag{95}$$

$$X_{ik} \geq 0 \tag{96}$$

Where:

W_{ik}	$= L_i(N_i + E_i) - (t_k + d_j + e_j + r_{ij})$ is waiting time associated with assigning truck k to shovel i
X_{ik}	is the decision on assigning truck k to ith shovel
S_k	is supply of truck k
D_i	is the demand of ith shovel
L_i	is the mean loading time of ith shovel
N_i	is the number of trucks at ith shovel
E_i	is the number of trucks en route to ith shovel
t_k	is the expected travel time of truck k to reach discharge point
d_j	is the expected waiting time of truck at discharge point j
c_j	is the average dumping time of truck at discharge point j
r_{ij}	is average empty travel time from discharge point j to ith loader

Eq.(94) ensures that total number of trucks assigned cannot exceed number of available trucks. Eq.(95) makes sure that trucks are sent to the ith shovel will cover its lost ton as much as possible. And, Eq.(96) ensures that number of type k trucks assigned to ith shovel is non-negative. The model assumes heterogeneous truck fleet, as a result it will be as close to reality as the upper stage model is. It also considers the situation a shovel is far behind its target production and needs to be assigned more than one truck. In such a situation the model easily assign more than a single truck to those needy shovel further behind the schedule without any limitation occurs by implementing assignment model. However, there are two major drawbacks with the model. The first major drawback is that mean of production rate for all routes is the basis for calculating the deviation of routes. But based on upper stage plan, sometimes it is required to extract much more of some specific materials that makes production rate of the routes of transporting those material be maximized and for some other be as less as possible. Then during the assignment it will send more trucks to those with higher negative deviation. The second major drawback is in transportation problem cost of transporting unit of material is constant and independent of supplier centers. Whereas, each truck waiting time at shovel or crusher is depending on the trucks previously assigned especially in over-truck systems. Also the waiting time accounting for in transportation method is based on trucks currently at destination or en route to the destination and there is no way to account for the waiting time will be caused by trucks will be assigned in the future but will reaches the destination earlier (Alarie and Gamache, 2002).

4.2.4. Single Stage Approach

In academic manners, one of the first algorithms introduced to solve truck allocation and dispatching problem in open pit mines is a single stage algorithm presented by Hauck (1973). The main feature of presented algorithm is combination of operation plan and real-time scheduling in a single model. The model is based on solving a sequence of assignment problem by using of DP. The model considers stripping ratio, blending objectives, capacity of the plant and stockpile. Objective of the model is to maximize the production by minimizing the lost ton caused by shovels idle time:

$$\min \sum_{j}\sum_{i}\sum_{q(j)=1}^{Q(j)} W_{ij}(t_{ij}(p(i),q(j)))X_{ij}(t_{ij}(p(i),q(j))) \tag{97}$$

Subject to:

$$r_L \le \frac{\sum_{l=1}^{k}\sum_{i}\sum_{j\in J_1} C_i X_{ij}(t_l)+a}{\sum_{l=1}^{k}\sum_{i}\sum_{j\in J_2} C_i X_{ij}(t_l)+b} \le r_U \qquad \textit{for each } k \tag{98}$$

$$\sum_{l=1}^{k}\sum_{i}\sum_{j\in J_2} C_i X_{ij}(t_l)+(V(t_0)-V(t_k)) \le R_U t_k \qquad \textit{for each } k \tag{99}$$

$$\sum_{l=1}^{k}\sum_{i}\sum_{j\in J_3} C_i X_{ij}(t_l)+(\sum_{l=1}^{k}\sum_{i}\sum_{j\in J_2} C_i X_{ij}(t_l)+(V(t_0)-V(t_k))) \ge R_L t_k \qquad \textit{for each } k \tag{100}$$

$$\sum_{i}\sum_{j\in J_3} C_i X_{ij}(t_k) \le V(t_k) \qquad \textit{for each } k \tag{101}$$

Where:

m	is number of available trucks
n	is number of shovels
C_i	is average haulage capacity of truck i
J_1	is the set of shovels j working at waste
J_2	is the set of shovels j working at ore mining faces
J_3	is the set of shovels j working at stockpile
t_k	is the time a shovel has just loaded a truck (assuming discrete points in time to keep track of the process)
Q(j)	is total number of loads completed by jth shovel in T working cycle
p(i)	is pth load of truck i
q(j)	is qth load of shovel j
$t_{ij}(p(i),q(j))$	is the earliest time pth load of truck i which is qth load of shovel j is loaded by shovel j on truck i

$$W_{ij}(t_{ij}(p(i),q(j))) = \begin{cases} E_j \Gamma_{ij}(t_{ij}(p(i),q(j))) & j \in J_{12} \\ 0 & j \in J_1 \end{cases}$$

E_j	is the loading rate of jth shovel (ton/time)
$\Gamma_{ij}(t_{ij}(p(i),q(j)))$	is the idle time incurred by jth shovel when it loads its q(j) load as truck’s p(i) load into the truck
r_L and r_U	are the lower and upper limits of SR
b	is a suitable quantity of ore

$a = b(r_L + r_U)/2$

R_L *and* R_U	are minimum and maximum processing plant rate
$V(t_o)$ and $V(t_k)$	are stockpile inventory at the beginning of the cycle and at time t_k

For each k^th decision an assignment problem is solved as a sub problem by implementing DP which has been presented in Eq.(77) to Eq.(80).

Eq.(98) ensures meeting SR requirement; Eqs.(99) and (100) guarantee that processing plant is always being fed; Eq.(101) ensures that total material handling at stockpile cannot exceed the amount of current stockpile inventory. D_k is assignment domain satisfying Constraints (99) to (101) also are the criteria defined for assignment in D_k domain. The algorithm presents optimal combinatorial intractable assignment procedure. Although it is a complex algorithm containing all limitation satisfaction criteria, it runs fast. However, assuming the problem as a completely deterministic procedure shows that stochastic properties of truck waiting time is ignored. Meeting all the production requirements is not the goal of the operation for each assignment and if they can be satisfied in a longer period of the time, their short term violation is acceptable. As previously be mentioned DP tries to find optimal solution from all of the feasible solutions of previous sub problems not from the best solution of them.

4.2.5. Some Other Efforts

Krause and Musingwini (2007) used machine repair analogy to analyze and choose truck fleet size for an open pit mine. They chose Arena for the simulation part "because it can be programmed with any number of probability distribution fitted to an unlimited number of cycle variables and is therefore a very flexible model for use in analyzing several variables in shovel-truck analysis".
The analogy is as follow: "The Machine Repair Model equivalents are shown in parenthesis. A truck is sent for loading (repair) every cycle with the number of shovels or shovel loading sides or number of tipping bins (repair bays) being equal to R and the inter-arrival and service times both assumed to have an exponential distribution. Therefore, a shovel-truck system can be described as M/M/R/GD/K/K, where the first M is truck arrival rate, the second M is loader service rate, R is the number of shovels or shovel loading sides that are loading K trucks drawn from a population of size K, whereby the loading follows some general queue discipline, GD".

He et al. (2010) implement GA to optimize truck dispatching problem in open pit mines. They tried to find a route and assign upcoming truck to it based on minimized transportation and maintenance costs. In that model it has been assumed that velocity of trucks in both loaded and empty condition are the same that is a drawback for their model. Although, their major focus was on minimizing the costs, by assuming same velocity for both loaded and unloaded trucks they underestimated costs. Another major drawback which is similar to almost all other models is assignment of trucks to routes not to shovel-destination. They claimed that truck maintenance cost get higher with the age of the truck by a constant coefficient, whereas Topal and Ramazan (2012) revealed that maintenance cost behaves in a fluctuated manner during its life and by each main repair the equipment's maintenance cost will decrease dramatically.

5. Limitations of Current Algorithms and Future Research Directions

5.1. Linking between strategic level and operational level plans

Many researchers and companies have been work on open pit mine operational planning, there are still many restrictions in the algorithms and models, though. The main objective in mining projects is to maximize the net present value (NPV). To achieve the main goal of the expenditure the operational plan has to be tied to both short-term and long-term plans. But with the current models of operation optimization there is no guarantee of meeting the main goal. Short-term production schedule which is the closest part of strategic planning to the operation planning provides destination of material from mining cuts, but in reality it will not be followed up to the minute.

5.2. Accounting for Uncertainty

Most of the models for operation optimization are deterministic and also current simulation models do not cover all the mine life. However, the nature of mining operation is stochastic and it is a multi-period task in which each period effects on later ones up until end of the mine life.

Beside the stochastic operation, material quality in each mining face is stochastic as well. But most of the models assume constant average grade for each mining face which causes lack of optimality.

5.3. Modeling Close to Reality

Although most mines are using heterogeneous trucks, mixed fleet is ignored in most of the dispatching models.

All models developed for mine operational plan optimization have been validated by using of a simulation model of an actual mine. In almost all the simulation models presented for validation of the models modeling the processing plant and hoppers have been ignored. To evaluate the models it is suggested that a simulation model of a complete open pit mine operation be used to be as close as possible to the reality.

5.4. Dynamic Best Path Determination

In small mines with a limited number of route segments and small fleet a fixed shortest path between all pair of loaders and destination is sufficient. However, for very large open pit mines there exist a vast network of haul roads and a large fleet of which trucks travel in the operation area. A large fleet of trucks usually consists of variety of truck types with different speed limits and averages which causes traffic mass on some route segments. Consequently, it will cause lost production. To fix this problem it is shortest paths can be determined dynamically. In the other word, by keeping track of trucks working in the system determine the shortest path between the current location of the truck and its next destination based on the time it will take to reach the destination regarding current traffic jam on approaching route segments.

5.5. Real-time Dispatching based on Transshipment Problem

In dispatching procedure it is recommended to implement transshipment problem instead of transportation or assignment approaches. In a transshipment problem in addition to supplier and demand points there exist transshipment points through which material can be transported from suppliers to demand points. In mining system stockpiles can be assumed as transshipment points.

6. Conclusion

Open pit mine operational plan algorithms and models first have been divided into two major classes of industrial and academic algorithms and then have been reviewed in this paper. The planning problem has been broken down to three major sub problems (1- finding the shortest paths, 2-operation optimization, and 3- real-time truck dispatching). Then for each sub problem existing algorithms have been reviewed. Limitation of current models introduced and suggestions for the future works presented.

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