

Oil Sands Integrated Mine Planning and Tailings Management

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Abstract

A strategic mine planning model determines the best order of extraction and destination of material over the mine-life, to maximize the net present value (NPV) of the produced minerals. In oil sands open-pit mining, further processing of the extracted oil sands generates massive volumes of tailings. To save space, the tailings are deposited in in-pit tailings containments constructed by internal dykes using mine waste material. In this paper, an integrated mine planning framework is proposed and implemented using mixed-integer linear programming to optimize the production schedule with respect to dyke construction and in-pit tailings deposition. A case study is carried out to verify the performance of the proposed optimization model. The results approves that the produced tailings is being deposited in the excavated mining-pit as the mining operations proceed and the in-pit dykes are constructed using mine waste material.

1. Introduction

An oil sands deposit is a mixture of bitumen and water in sands and clay. It is a thick, sticky, heavy and viscous material and needs rigorous extraction treatment to refine its bitumen. The oil sands is one of the fastest growing industries in North America. Though in recent times oil prices are relatively low, there have been considerable investments in the past that can keep this industry vibrant for some decades. It is also more relevant now that further research aimed at improving the profitability of these operations in the long-term is pursued.

A mine production plan determines the best schedule for extraction and the destination of the extracted material, in a way that maximizes the net present value of the production. In oil sands operations, the material mined is sent to the processing plant for extraction of bitumen through hot water extraction process, which produces tailings. About 80% of the material sent to the processing plant ends up in the tailings dam.

Solid waste management is a related concept to long-term mine planning. Mining operations generate considerable volumes of solid waste mostly as overburden and interburden (OI) to access the mineralized zone. The current practice is to dump the waste material for later use mostly in dyke construction and reclamation. The dykes may be constructed either in-pit or ex-pit depending on the waste management strategy in place at the time. The main source of the required material for dyke construction is OI material coming from mining operations, and the tailings coarse sand (TCS)

coming from processing plant (Ben-Awuah, 2013, Fauquier et al., 2009). Ben-Awuah et al. (2012) provide a detailed description of an integrated oil sands mining operation including material flows, solid waste and tailings management. Hence, waste disposal, reclamation planning and dyke construction planning can be integrated with the mine planning framework. In the literature, few works have addressed such integration, but none of them has covered the mentioned domains completely (Badiozamani and Askari-Nasab, 2013, Badiozamani and Askari-Nasab, 2014a, Badiozamani and Askari-Nasab, 2014b, Ben-Awuah, 2013, Ben-Awuah and Askari-Nasab, 2011, Ben-Awuah et al., 2012).

Since 1960s, operations research techniques such as linear programming, integer programming, mixed-integer linear programming (Johnson, 1969) and dynamic programming (Tan and Ramani, 1992) have been used to find the optimized pattern of extraction and determine a destination for the extracted material in open-pit mining and block caving (Newman et al., 2010). The most common way to control the precedence order of extraction for mining blocks is to define integer variables, which makes the mine planning problem a non-deterministic polynomial-time hard for large-scale problems (Gleixner, 2008). Due to the large number of integer variables corresponding to mining blocks over large number of periods, it takes considerably a long time for the current solvers to solve the problem.

The mixed-integer linear programming (MILP) is a powerful tool extensively used in the literature for mine planning optimization. Typical mine planning models maximize the NPV over the mine-life, with respect to the mining and processing capacities, ore blending constraints, and spatial precedence among mining blocks (Askari-Nasab and Awuah-Offei, 2009, Askari-Nasab et al., 2011b, Johnson, 1969). Further than the pure long-term mine planning models, few works are published addressing the linkage between mine planning and tailings production (Kalantari et al., 2013). The solid waste disposal management and dyke construction planning in oil sands are also integrated into the long-term mine planning framework (Ben-Awuah, 2013, Ben-Awuah and Askari-Nasab, 2011, Ben-Awuah et al., 2012).

Badiozamani and Askari-Nasab (2014a) proposed an integrated model for long-term mine planning, with respect to reclamation material handling and tailings capacity constraints. The concept of directional mining is used in modeling to provide capacity for in-pit tailings facility. The model determines the destination for each extracted parcel (dynamic cut-off grade) in such a way to maximize the NPV over the mine-life. Mining aggregates are used in the model to follow the selective mining units. The authors reach to integer solutions within 2% optimality gap in less than 10 minutes for the cases with more than 98,000 mining-blocks aggregated to 535 mining-cuts. The optimality gap refers to the absolute tolerance on the gap between the best integer objective and the objective of the best node remaining in the branch and cut algorithm. The resulting schedule generates the maximum NPV, minimizes the material handling cost of reclamation, and the tailings volume produced downstream meets the tailings capacity constraints in each period. The authors take a further step in integrated mine planning, by including the tailings management in terms of composite tailings (CT) production and deposition, in the mine planning optimization framework (Badiozamani and Askari-Nasab, 2014b).

The gap in current literature is the integration of all these areas: maximization of profit in pure mine planning, minimization of dyke construction costs, and minimization of tailings disposal costs. The proposed MILP model in this research maximizes the net present value (NPV) and at the same time minimizes the costs of dyke construction and CT deposition. The optimization is subject to a number of constraints, including the mining, processing, and tailings storage capacities and extraction precedence constraints. This integrated model will reduce the re-handling cost of dyke construction and reclamation material. The model schedules these material types when they are needed both in quality and quantity directly to the appropriate destination.

2. The mathematical model

The proposed mathematical model includes both tailings management, in terms of CT deposition, and waste management in terms of dyke construction planning. The objective function includes three parts as: (1) maximization of NPV, (2) minimization of Dyke construction costs, and (3) minimization of CT deposition costs. Mining-panels (intersections of bench faces and pushbacks) are used as the units for mining operations, while mining-cuts (aggregated blocks) are used for processing. The detailed structure of the MILP model is as follows:

2.1. Objective function

The objective function maximizes the net present value of the profit gained from processing of each mining-panel. The revenue from each mining-panel consists of two terms: the revenue from selling each tonne of bitumen, and a summation of operational costs. The operational costs include the material extraction costs, the extra costs for mining ore material, and the cost of selling the ore. The other two operational costs are the extra costs of mining and preparing material for dyke construction, and the cost of CT deposition in the CT cells.

The economic value of mining panels is calculated through Eqs. (1) to (5):

$$d_p^{a,u,t} = \sum_{k \in p_a} (r_k^{u,t} - n_k^{u,t} - m_k^{u,t}) - q_p^{a,t} \quad \forall t \in \mathbf{T}, u \in \mathbf{U}, p \in \mathbf{P}, a \in \mathbf{A} \quad (1)$$

Where:

$$r_k^{u,t} = \sum_{e=1}^E o_k \times g_k^e \times r^{u,e} \times (p^{e,t} - cs^{e,t}) - \sum_{e=1}^E o_k \times cp^{u,e,t} \quad \forall t \in \mathbf{T}, u \in \mathbf{U}, k \in \mathbf{K} \quad (2)$$

$$q_p^{a,t} = \sum_{k \in p} (o_k + d_k + w_k) \times cm^{a,t} \quad \forall t \in \mathbf{T}, p \in \mathbf{P}, a \in \mathbf{A} \quad (3)$$

$$n_k^{u,t} = d_k \times cl^{u,t} \quad \forall t \in \mathbf{T}, u \in \mathbf{U}, k \in \mathbf{K} \quad (4)$$

$$m_k^{u,t} = l_k \times cu^{u,t} \quad \forall t \in \mathbf{T}, u \in \mathbf{U}, k \in \mathbf{K} \quad (5)$$

And the cost of CT deposition is calculated as in Eq. (6):

$$i_c^t = h_c \times ct^{c,t} \quad \forall t \in \mathbf{T}, c \in \mathbf{C} \quad (6)$$

The objective function is defined as Equation (7):

$$\text{Max} \sum_{t=1}^T \left(\sum_{u=1}^U \sum_{a=1}^A \sum_{j=1}^J \sum_{\substack{p \in B_j \\ k \in B_p}} [r_k^{u,t} \times x_k^{u,t} - q_p^t \times y_p^{a,t} - (n_k^{u,t} \times w_k^{u,t} + m_k^{u,t} \times v_k^{u,t})] - \sum_{c=1}^C i_c^t \times z_c^t \right) \quad (7)$$

2.2. Constraints

The optimization is subject to the constraints stated by Eqs. (8) to (45). Eqs. (8) and (9) present the mining and processing capacity constraints. Eqs. (10) and (11) ensure that the material sent for dyke construction are within the range of minimum and maximum requirements. Eqs. (12), (13) and (14) control the balance of material tonnages extracted and used for different purposes. The blending

constraints for ore and OI material are presented in Eqs. (15), (16), and (17). Eqs. (18), (19), (20) and (21) add up the total tonnage of different components of tailings and ensure that the tonnages are not exceeding the corresponding capacity ranges. Eqs. (22), (23) and (24) ensure that the total CT produced and deposited in CT cells does not exceed the capacity of CT containments in each period. Mining precedence constraints are presented in Eqs. (25) to (31). The precedence order of CT cells construction and CT deposition is controlled through Eqs. (32) to (39). Finally, Eqs. (40) to (45) ensure that the summation of decision variables adds up to one.

$$T_{Mi}^{a,t} \leq \sum_{j=1}^J \left(\sum_{p \in B_j} \sum_{k \in B_p} (o_k + w_k + d_k) \times y_p^{a,t} \right) \leq T_{Mu}^{a,t} \quad \forall t \in \mathbf{T}, \forall a \in \mathbf{A} \quad (8)$$

$$T_{Pi}^{u,t} \leq \sum_{j=1}^J \left(\sum_{k \in B_j} (o_k \times x_k^{u,t}) \right) \leq T_{Pu}^{u,t} \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (9)$$

$$T_{Ci}^{u,t} \leq \sum_{j=1}^J \left(\sum_{k \in B_j} (d_k \times w_k^{u,t}) \right) \leq T_{Cu}^{u,t} \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (10)$$

$$T_{Ni}^{u,t} \leq \sum_{j=1}^J \left(\sum_{k \in B_j} (l_k \times v_k^{u,t}) \right) \leq T_{Nu}^{u,t} \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (11)$$

$$\sum_{u=1}^U \sum_{k \in B_p} (o_k \times x_k^{u,t} + d_k \times w_k^{u,t}) \leq \sum_{a=1}^A \sum_{k \in B_p} (o_k + d_k) \times y_p^{a,t} \quad \forall t \in \mathbf{T}, p \in \mathbf{P} \quad (12)$$

$$\sum_{u=1}^U (l_k \times v_k^{u,t}) \leq \sum_{u=1}^U (o_k \times x_k^{u,t}) \quad \forall t \in \mathbf{T}, k \in \mathbf{K} \quad (13)$$

$$\sum_{d=1}^D (k_d \times u_d^t) \leq \sum_{u=1}^U \sum_{j=1}^J \left(\sum_{k \in B_j} (d_k \times w_k^{u,t} + l_k \times v_k^{u,t}) \right) \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (14)$$

$$\underline{g}^{u,t,e} \leq \sum_{j=1}^J \left(\sum_{k \in B_j} g_k^e \times o_k \times x_k^{u,t} / \sum_{k \in B_j} o_k \times x_k^{u,t} \right) \leq \overline{g}^{u,t,e} \quad \forall t \in \mathbf{T}, u \in \mathbf{U}, e \in \mathbf{E} \quad (15)$$

$$\underline{f}^{u,t,o} \leq \sum_{j=1}^J \left(\sum_{k \in B_j} f_k^o \times o_k \times x_k^{u,t} / \sum_{k \in B_j} o_k \times x_k^{u,t} \right) \leq \overline{f}^{u,t,o} \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (16)$$

$$\underline{f}^{u,t,c} \leq \sum_{j=1}^J \left(\sum_{k \in B_j} f_k^c \times d_k \times w_k^{u,t} / \sum_{k \in B_j} d_k \times w_k^{u,t} \right) \leq \overline{f}^{u,t,c} \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (17)$$

$$T_{Tl}^{u,t} \leq \sum_{j=1}^J \left(\sum_{k \in B_j} (t_k \times x_k^{u,t}) \right) \leq T_{Tu}^{u,t} \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (18)$$

$$T_{Fl}^{u,t} \leq \sum_{j=1}^J \left(\sum_{k \in B_j} (f_k \times x_k^{u,t}) \right) \leq T_{Fu}^{u,t} \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (19)$$

$$T_{Sl}^{u,t} \leq \sum_{j=1}^J \left(\sum_{k \in B_j} (s_k \times x_k^{u,t}) \right) \leq T_{Su}^{u,t} \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (20)$$

$$T_{Wl}^{u,t} \leq \sum_{j=1}^J \left(\sum_{k \in B_j} (r_k \times x_k^{u,t}) \right) \leq T_{Wu}^{u,t} \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (21)$$

$$T_{Xl}^{u,t} \leq \sum_{j=1}^J \left(\sum_{k \in B_j} (h_k \times x_k^{u,t}) \right) \leq T_{Xu}^{u,t} \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (22)$$

$$T_{Yl}^{u,t} \leq \sum_{j=1}^J \left(\sum_{k \in B_j} (p_k \times x_k^{u,t}) \right) \leq T_{Yu}^{u,t} \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (23)$$

$$\sum_{c=1}^C (h_c \times z_c^t) \leq \sum_{j=1}^J \sum_{k \in B_j} \sum_{u=1}^U (p_k \times x_k^{u,t}) \quad \forall t \in \mathbf{T} \quad (24)$$

$$b_p^t - \sum_{a=1}^A \sum_{i=1}^t y_s^{a,i} \leq 0 \quad \forall t \in \mathbf{T}, p \in \mathbf{P}, s \in N_p(L) \quad (25)$$

$$b_p^t - \sum_{a=1}^A \sum_{i=1}^t y_r^{a,i} \leq 0 \quad \forall t \in \mathbf{T}, p \in \mathbf{P}, r \in O_p(L) \quad (26)$$

$$\sum_{a=1}^A \sum_{i=1}^t y_p^{a,i} - b_p^t \leq 0 \quad \forall t \in \mathbf{T}, p \in \mathbf{P} \quad (27)$$

$$b_p^t - b_p^{t+1} \leq 0 \quad \forall t \in \{1, \dots, T-1\}, p \in \mathbf{P} \quad (28)$$

$$H \times c_j^t - \sum_{a=1}^A \sum_{i=1}^t y_h^{a,i} \leq 0 \quad \forall t \in \mathbf{T}, j \in \mathbf{J}, h \in B_j(H) \quad (29)$$

$$\sum_{a=1}^A \sum_{i=1}^t y_h^{a,i} - H \times c_j^t \leq 0 \quad \forall t \in \mathbf{T}, j \in \mathbf{J}, h \in B_{j+1}(H) \quad (30)$$

$$c_j^t - c_j^{t+1} \leq 0 \quad \forall t \in \{1, \dots, T-1\}, j \in \mathbf{J} \quad (31)$$

$$a_c^t - \sum_{i=1}^t z_r^i \leq 0 \quad \forall t \in \mathbf{T}, c \in \mathbf{C}, r \in Q_c(R) \quad (32)$$

$$\sum_{i=1}^t z_c^i - a_c^t \leq 0 \quad \forall t \in \mathbf{T}, c \in \mathbf{C} \quad (33)$$

$$a_c^t - a_c^{t+1} \leq 0 \quad \forall t \in \{1, \dots, T-1\}, c \in \mathbf{C} \quad (34)$$

$$q_d^t - \sum_{i=1}^t u_m^i \leq 0 \quad \forall t \in \mathbf{T}, d \in \mathbf{D}, m \in S_d(G) \quad (35)$$

$$\sum_{i=1}^t u_d^i - q_d^t \leq 0 \quad \forall t \in \mathbf{T}, d \in \mathbf{D} \quad (36)$$

$$q_d^t - q_d^{t+1} \leq 0 \quad \forall t \in \{1, \dots, T-1\}, d \in \mathbf{D} \quad (37)$$

$$q_d^t - \sum_{a=1}^A \sum_{i=1}^t y_f^{a,i} \leq 0 \quad \forall t \in \mathbf{T}, d \in \mathbf{D}, f \in X_d(P) \quad (38)$$

$$a_c^t - \sum_{i=1}^t u_n^i \leq 0 \quad \forall t \in \mathbf{T}, c \in \mathbf{C}, n \in T_c(D) \quad (39)$$

$$\sum_{u=1}^U \sum_{t=1}^T x_k^{u,t} \leq 1 \quad \forall k \in \mathbf{K} \quad (40)$$

$$\sum_{u=1}^U \sum_{t=1}^T w_k^{u,t} \leq 1 \quad \forall k \in \mathbf{K} \quad (41)$$

$$\sum_{u=1}^U \sum_{t=1}^T v_k^{u,t} \leq 1 \quad \forall k \in \mathbf{K} \quad (42)$$

$$\sum_{t=1}^T z_c^t \leq 1 \quad \forall c \in \mathbf{C} \quad (43)$$

$$\sum_{t=1}^T u_d^t \leq 1 \quad \forall d \in \mathbf{D} \quad (44)$$

$$\sum_{t=1}^T y_p^{a,t} \leq 1 \quad \forall p \in \mathbf{P}, a \in \mathbf{A} \quad (45)$$

3. Case study: mine planning with consideration of dyke construction and composite tailings (CT) deposition

This case study is designed to show how tailings management, in terms of CT production, and the solid waste management, in terms of dyke construction can be integrated in the production schedule and how the overall NPV is sensitive to such an integration. The specification of the material contained in the block model, which is the input to Whittle for pit-limit optimization is presented in Table 1. The parameters used for optimization in the case study are presented in Table 2. The mineralized material is defined by a regulatory cut-off grade of 7% bitumen content; and the cut-off size between fines and coarse sand is 44 μm (Masliyah, 2010). The overall stripping ratio for this deposit is 2.3:1.

Table 1. Specifications of the block model used in Whittle for pit-limit optimization

Block model data	Value
Devonian rock type	4,526 Mt
McMurray Formation (MMF)	642 Mt
Overburden	446 Mt
Bitumen Content in MMF	54 Mt
Average Bitumen Grade in MMF	8%
Fines Content in MMF	86 Mt
Average Fines Grade in MMF	13%
Water Content in MMF (Mt)	26 Mt

Table 2. MILP Input parameters used in case study

Input parameters	Value	Input parameters	Value
Recovered barrel of bitumen per tonne of Bit.	0.65	Extra OI dyke mining cost (\$/t)	0.92
Ore Price (\$/t of Bitumen)	450	Extra TCS dyke mining cost (\$/t)	1.38
Mining Cost (\$/t)	4.60	CT deposition cost (\$/m ³)	0.50
Processing Cost (\$/t)	5.03	Ore cut-off grade	7%
Total material (Mt)	1,237	Upper bound on fines grade in ore	18%
Mineralized material (Mt)	374	Upper bound on fines grade in OI	30%
OI material (Mt)	597	Interest rate	10%
TCS material (Mt)	278	Recovery	90%
Mining direction	W-E	Number of mining-panels	70
Number of periods (years)	10	Number of mining-cuts	972

In the proposed model, it is assumed that the produced CT will be deposited mainly in a number of in-pit CT cells shaped by internal dykes and pit walls. The external tailings facility (ETF) also acts

as a buffer to accommodate the excess of the mature fine tailings (MFT) when the CT production has not yet started or when the internal dykes are not available. The internal dykes ensure that both mining and tailings deposition can occur simultaneously in the pit during the mine life.

Before raising the internal dyke walls, the first step is to choose the dykes' footprints. To guarantee a feasible schedule, dyke footprints are selected from among pushback footprints. This selection is made based on the volume of material in pushbacks and the potential volume of CT to be produced from processing the extracted material. Since the pushbacks are extracted following a precedence order, no material will be left behind before constructing a dyke, and the dyke footprint has been cleared already. Fig. 1 illustrates a plan view of the dyke footprints and the schematic ETF used in the case study. The in-pit colors represent the mining panels used to control material extraction on this level.

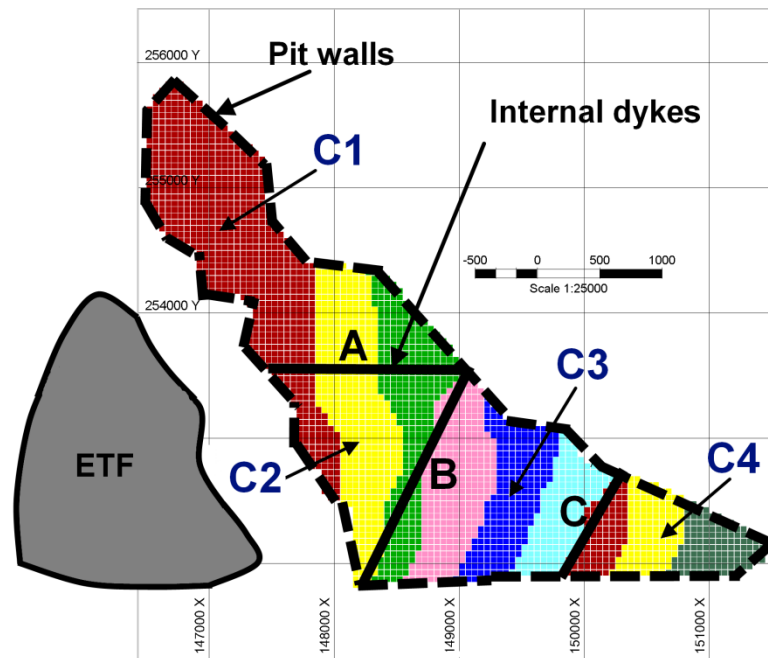


Fig. 1. Dykes' footprints (A & B), CT cells (C1 to C4), and the ETF for the case study

4. Discussion of results

In this case study, the ETF works only as a buffer, and since it has a limited capacity, the in-pit CT cells must be prepared for CT storage. In order to meet such a requirement, the OI and TCS material must be produced and used for the construction of in-pit dykes. Solving the MILP generates an NPV of \$3,959M over 10 years. It has resulted in the extraction of 1,237 Mt of material, including 314 Mt of mineralized material, 264 Mt of OI, and 659 Mt of waste (Table 3).

Processing the mineralized material generates 37 Mt of bitumen and 92 Mt of TCS. A total of 227 Mm³ of CT is produced, from which 148 Mm³ (65%) is deposited in the ETF and the rest (79 Mt) is deposited in the in-pit CT cell C1. The total material usable for dyke construction is 356 Mt (OI and TCS), from which 159 Mt is used to construct Dyke A and the rest are sent to the waste dump. The resulting production schedule is presented in

Fig. 2. The production schedule generated ensures a uniform mill feed and OI material for dyke construction. There is however some fluctuations in the waste material mined which may require contract mining or equipment lease options during certain periods to ensure efficient utilization of the owner mining fleet.

Table 3. Numerical results of the case study

Total material extracted	Mineralized material Extracted	Processed ore	Recovered bit.	Extracted OI
1,237 Mt	374 Mt	313.7 Mt	37.03 Mt	264 Mt
Produced TCS	Optimality Gap	Run time	NPV	
92 Mt	0.0%	63 s	3,959 M\$	

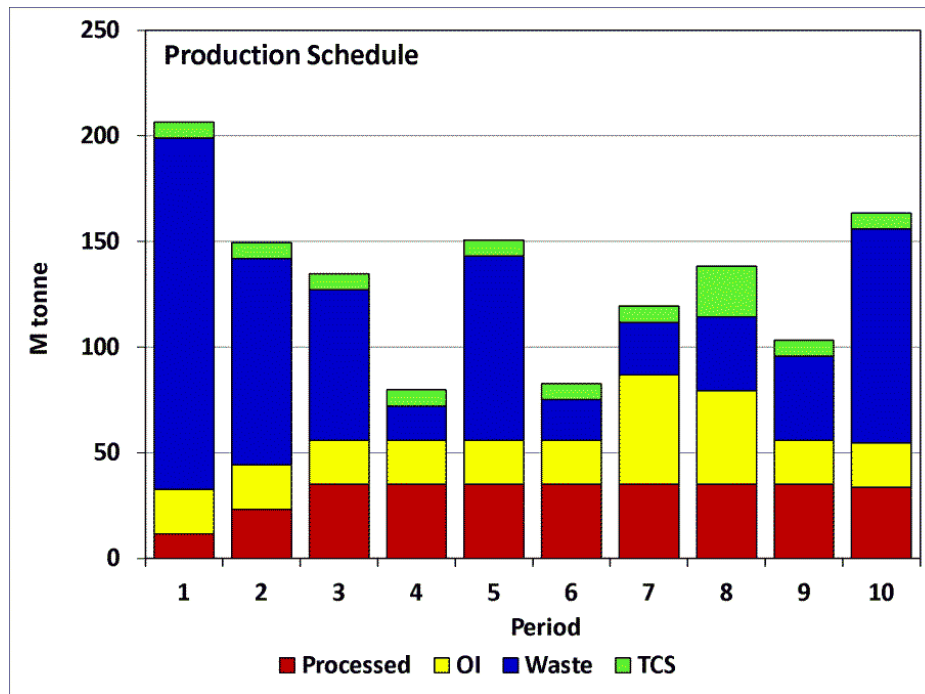


Fig. 2. Production schedule

Fig. 3 illustrates how the nine pushbacks are extracted. Pushback mining follows the West-to-East direction, and the generated schedule ensures that before mining starts in one pushback, the previous pushback has already been extracted. In this way, the footprint of Dyke A as the first in-pit dyke is cleared after pushback three has been completely extracted (in the sixth period).

The only in-pit dyke being constructed is Dyke A and its construction begins after pushback three has been completely extracted in period six (Fig. 4). During periods one to seven when the in-pit cell is not yet ready for tailings deposition, the produced CT is sent to the ETF, as illustrated in Fig. 5. After CT cell 1 is completed in period 8, the CT is deposited in this CT cell over periods 8 to 10.

Fig. 6 illustrates the periods in which the eight lifts of Dyke A are constructed, as well as the start and end periods of CT deposition in the ETF and in CT cell 1. Implementation of the MILP model generates an NPV of \$3,959M.

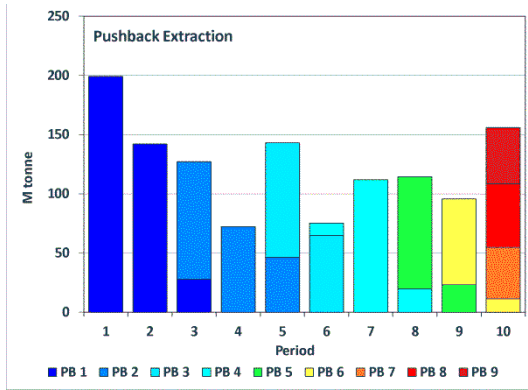


Fig. 3. Pushback extraction schedule

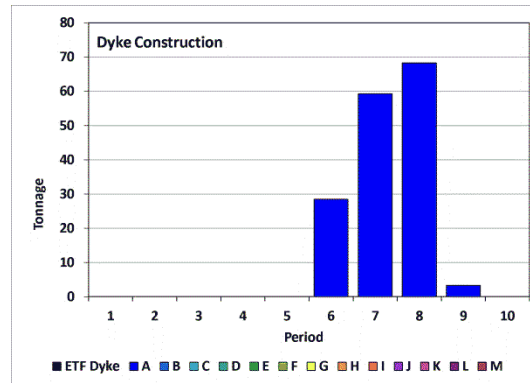


Fig. 4. Dyke construction schedule

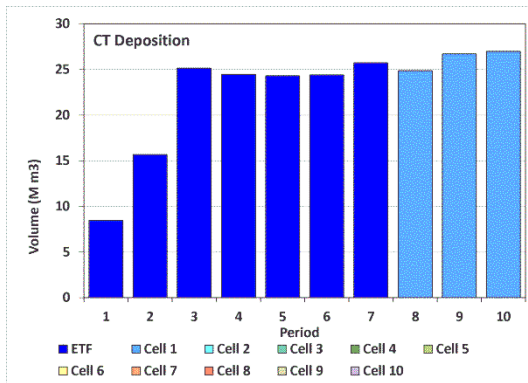


Fig. 5. CT deposition schedule

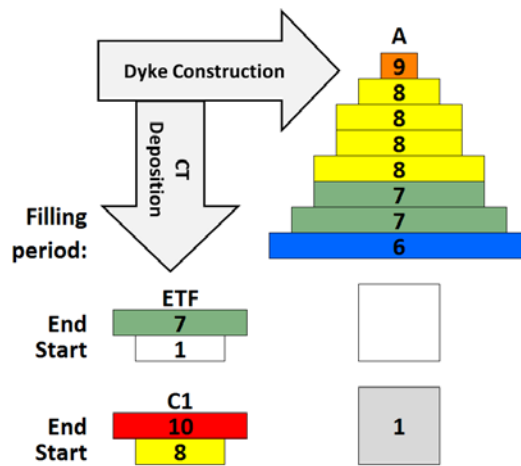


Fig. 6. Construction of Dyke A and CT deposition in the ETF and CT cell

5. Conclusions

The literature related to mine planning and waste management is reviewed. The current literature lacks integration between mine planning and waste management in terms of in-pit deposition of solid waste material and tailings. The implemented framework is a novel topic that fills the current literature gap in strategic open-pit mine planning. An integrated long-term mine production plan has been developed to solve the optimal mine production schedule, with respect to dyke construction and tailings deposition. The model is verified through a case study on a real oil sands data set. The generated schedule is practically mineable, follows the chosen direction, provides a smooth feed for the oil sands processing plant, provides the material required to construct in-pit dykes, and accommodates the produced CT in the ETF and in-pit CT cells. The value of this model to the mining industry can be quantified directly from the savings made by avoiding re-handling of the dyke construction material and indirectly from the reduced mining footprint.

It is recommended to consider efficient methods to reduce the problem size for large-scale problems, through preprocessing and period aggregation technique. The other area for development of the research is to consider other means of tailings dewatering, such as atmospheric fine drying (AFD) used in ETFs, or non-segregated tailings technology (NST) for in-pit impoundment of tailings products.

6. References

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7. Appendix

7.1. Decision variables

- $x_k^{u,t} \in [0,1]$ A continuous variable representing the portion of ore from mining-cut k to be extracted and processed at destination u in period t.
- $w_k^{u,t} \in [0,1]$ A continuous variable representing the portion of OI material from mining-cut k to be extracted and used for dyke construction at destination u in period t.
- $v_k^{u,t} \in [0,1]$ A continuous variable representing the portion of TCS from mining-cut k to be extracted and used for dyke construction at destination u in period t.
- $y_p^{a,t} \in [0,1]$ A continuous variable representing the portion of mining-panel p to be mined in period t from location a, which includes ore, OI material, tailings sand and waste.
- $z_c^t \in [0,1]$ A continuous variable representing the portion of CT cell c to be filled with CT in period t.
- $u_d^t \in [0,1]$ A continuous variable representing the portion of dyke unit d to be constructed in period t.
- $b_p^t \in \{0,1\}$ A binary integer variable controlling the precedence of extraction of mining-panels. b_p^t is equal to one if the extraction of mining-panel p has started by or in period t, otherwise it is zero.
- $c_j^t \in \{0,1\}$ A binary integer variable controlling the precedence of mining phases. c_j^t is equal to one if the extraction of phase j has started by or in period t, otherwise it is zero.
- $a_c^t \in \{0,1\}$ A binary integer variable controlling the precedence of filling of CT cells. a_c^t is equal to one if the filling of CT cell c has started by or in period t, otherwise it is zero.
- $q_d^t \in \{0,1\}$ A binary integer variable controlling the precedence of Constructing dyke units. q_d^t is equal to one if the construction of dyke unit d has started by or in period t, otherwise it is zero.

7.2. MILP sets and indices

- $a \in \mathbf{A}, \mathbf{A} = \{1, \dots, A\}$ Index and set of all the possible mining locations (pits) in the model.
- $c \in \mathbf{C}, \mathbf{C} = \{1, \dots, C\}$ Index and set of all CT cells in the model.
- $d \in \mathbf{D}, \mathbf{D} = \{1, \dots, D\}$ Index and set of all dyke units in the model.
- $e \in \mathbf{E}, \mathbf{E} = \{1, \dots, E\}$ Index and set of all the elements of interest in the model.
- $j \in \mathbf{J}, \mathbf{J} = \{1, \dots, J\}$ Index and set of all the phases (push-backs) in the model.

- $k \in \mathbf{K}, \mathbf{K} = \{1, \dots, K\}$ Index and set of all the mining-cuts in the model.
- $p \in \mathbf{P}, \mathbf{P} = \{1, \dots, P\}$ Index and set of all the mining panels in the model.
- $t \in \mathbf{T}, \mathbf{T} = \{1, \dots, T\}$ Index and set of all the scheduling periods in the model.
- $u \in \mathbf{U}, \mathbf{U} = \{1, \dots, U\}$ Index and set of all the possible destinations for materials in the model.
- $B_j(H)$ For each phase j , there is a set $B_j(H) \subset \mathbf{P}$ defining the mining panels within the immediate predecessor pit phases (push-backs) that must be extracted prior to extracting phase j , where H is an integer number representing the total number of mining panels in the set $B_j(H)$.
- $B_p(V)$ For each mining panel p , there is a set $B_p(V) \subset \mathbf{K}$ defining the mining-cuts that belongs to the mining panel p , where V is the total number of mining-cuts in the set $B_p(V)$.
- $N_p(L)$ For each mining panel p , there is a set $N_p(L) \subset \mathbf{P}$ defining the immediate predecessor mining panels above mining panel p that must be extracted prior to extraction of mining panel p , where L is the total number of mining panels in the set $N_p(L)$.
- $O_p(L)$ For each mining panel p , there is a set $O_p(L) \subset \mathbf{P}$ defining the immediate predecessor mining panels in a specified horizontal mining direction that must be extracted prior to extraction of mining panel p at the specified level, where P is the total number of mining panels in the set $O_p(L)$.
- $Q_c(R)$ For each CT cell c , there is a set $Q_c(R) \subset \mathbf{C}$ defining the immediate predecessor CT cells below the CT cell c that must be filled in prior to filling of CT cell c , where R is the total number of CT cells in the set $Q_c(R)$.
- $S_d(G)$ For each dyke unit d , there is a set $S_d(G) \subset \mathbf{D}$ defining the immediate predecessor dyke units that must be constructed in prior to constructing of dyke cell d , where G is the total number of dyke units in the set $S_d(G)$.
- $T_c(D)$ For each CT cell c , there is a set $T_c(D) \subset \mathbf{D}$ defining the immediate predecessor dyke units that must be constructed in prior to filling of CT cell c , where D is the total number of dyke units in the set $T_c(D)$.
- $X_d(P)$ For each dyke unit d , there is a set $X_d(P) \subset \mathbf{P}$ defining the immediate predecessor mining panels that must be extracted in prior to construction of dyke unit d to guarantee that the dykes foot print is cleared, where P is the total number of panels in the set $X_d(P)$.

7.3. MILP parameters

- $cl^{u,t}$ Extra cost in present value terms for mining, shipping, and using a tonne of OI material for dyke construction at destination u .
- $cm^{a,t}$ Cost in present value terms of mining a tonne of waste in period t from mine a .
- $cp^{u,e,t}$ Discounted extra cost for mining and processing one tonne of ore at destination u .
- $cs^{e,t}$ Selling cost of element e in present value terms per unit of product.

$ct^{c,t}$	Cost in present value terms of sending a volume unite of CT in period t to cell c.
$cu^{u,t}$	Extra cost in present value terms for mining, shipping, and using a tonne of tailings sand for dyke construction at destination u.
$d_p^{a,u,t}$	Discounted profit obtained by extracting mining panel p from location a and sending it to destination u in period t.
d_k	OI dyke material tonnage in mining-cut k.
f_k	Fines tonnage produced from extracting all of the ore from mining-cut k.
f_k^c	Average percentage of fines in the OI dyke material portion of mining-cut k.
$\underline{f}^{u,t,c}$	Lower bound on the required average fines percentage of OI dyke material in period t at destination u.
$\overline{f}^{u,t,c}$	Upper bound on the required average fines percentage of OI dyke material in period t at destination u.
f_k^o	Average percentage of fines in the ore portion of mining-cut k.
$\underline{f}^{u,t,o}$	Lower bound on the required average fines percentage of ore in period t at processing destination u.
$\overline{f}^{u,t,o}$	Upper bound on the required average fines percentage of ore in period t at processing destination u.
g_k^e	Average grade of element e in the ore portion of mining-cut k.
$\underline{g}^{u,t,e}$	Lower bound on the required average head grade of element e in period t at processing destination u.
$\overline{g}^{u,t,e}$	Upper bound on the required average head grade of element e in period t at processing destination u.
h_c	Total volume of CT cell c.
h_k	MFT volume produced from extracting all of the ore from mining-cut k.
k_d	Total volume of dyke unit d.
l_k	Tailings coarse sand tonnage in mining-cut k.
$m_k^{u,t}$	Extra discounted cost of producing tailings sand from mining-cut k in period t and sending it for dyke construction in destination u.
$n_k^{u,t}$	Extra discounted cost of mining the OI material of the mining-cut k in period t and sending it for dyke construction in destination u.
o_k	Ore tonnage in mining-cut k.
$p^{e,t}$	Price of element e in present value terms per unit of product.
p_k	CT volume produced from extracting all of the ore from mining-cut k.

p_p	Mining panel p.
$q_p^{a,t}$	Discounted cost of mining all the material in mining panel p in period t as waste from location a.
r_k	Water tonnage produced from extracting all of the ore from mining-cut k.
$r^{u,e}$	Proportion of element e recovered if it is processed at destination u.
$r_k^{u,t}$	Discounted revenue obtained by selling the final products within mining-cut k in period t if it is sent to destination u, minus the extra discounted cost of mining all the material in mining-cut k as ore from location a and processing at destination u.
s_k	Sand tonnage produced from extracting all of the ore from mining-cut k.
t_k	Tailings tonnage produced from extracting all of the ore in mining-cut k.
$T_{Mu}^{a,t}$	Upper bound on mining capacity (tonnes) in period t at location a.
$T_{Ml}^{a,t}$	Lower bound on mining capacity (tonnes) in period t at location a.
$T_{Pu}^{u,t}$	Upper bound on processing capacity (tonnes) in period t at destination u.
$T_{Pl}^{u,t}$	Lower bound on processing capacity (tonnes) in period t at destination u.
$T_{Cu}^{u,t}$	Upper bound on OI material required for dyke construction (tonnes) in period t at destination u.
$T_{Cl}^{u,t}$	Lower bound on OI material required for dyke construction (tonnes) in period t at destination u.
$T_{Nu}^{u,t}$	Upper bound on tailings sand required for dyke construction (tonnes) in period t at destination u.
$T_{Nl}^{u,t}$	Lower bound on TCS required for dyke construction (tonnes) in period t at destination u.
$T_{Tu}^{u,t}$	Upper bound on capacity of tailings facility (tonnes) in period t at destination u.
$T_{Tl}^{u,t}$	Lower bound on capacity of tailings facility (tonnes) in period t at destination u.
$T_{Fu}^{u,t}$	Upper bound on capacity of fine material (tonnes) in period t at destination u.
$T_{Fl}^{u,t}$	Lower bound on capacity of fine material (tonnes) in period t at destination u.
$T_{Su}^{u,t}$	Upper bound on capacity of tailings sand (tonnes) in period t at destination u.
$T_{Sl}^{u,t}$	Lower bound on capacity of tailings sand (tonnes) in period t at destination u.
$T_{Wu}^{u,t}$	Upper bound on capacity of tailings water (tonnes) in period t at destination u.
$T_{Wl}^{u,t}$	Lower bound on capacity of tailings water (tonnes) in period t at destination u.
$T_{Xu}^{u,t}$	Upper bound on capacity of MFT (tonnes) in period t at destination u.
$T_{Xl}^{u,t}$	Lower bound on capacity of MFT (tonnes) in period t at destination u.

$T_{Y_u}^{u,t}$ Upper bound on capacity of CT (tonnes) in period t at destination u.

$T_{Y_l}^{u,t}$ Lower bound on capacity of CT (tonnes) in period t at destination u.

w_k Waste tonnage in mining-cut k.