Stochastic Long-Term Mine Production Scheduling

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Abstract

Mining is one of capital-intensive industries with high exploration, exploitation, and processing operating costs in order of hundreds of millions of dollars. Thus, mining projects should be planned with care to avoid possible losses. Generally, mine production planning of open pit mines cannot lead to a perfect plan in terms of mine management's goals if it does not consider uncertainties present in the mining operations. In this paper, a long-term mine production scheduling model is proposed to deal with the grade uncertainty present in production planning. the proposed model is applied regarding a number of realizations and generates a final schedule and profit as the most probable schedule and profit, respectively.

1. Introduction

Long-term planning of mining projects is a critically important part of open pit mining ventures and deals with the efficient management of cash flows in the order of hundreds of millions of dollars (Leite and Dimitrakopoulos, 2007). The main traditional objective of long-term mine scheduling is the maximization of cash flow or the net present value (NPV). However, various factors of risk in mining operations create uncertainties in the schedule and may lead to its failure. Two major concerns of investors and shareholders are the payback period and the risk associated with it. Uncertainties will endanger the mine schedule and finally the forecasted cash flows. Thus, consideration of risk in the schedule will show a more realistic plan to the mine's managers and investors. Uncertainties in the mining projects often originate from sources such as price volatility, geological uncertainty, economical instability, and different uncertainties in availability and utilization of mining and processing facilities. Each factor of risk can influence positively or negatively on the cash flow of a mining project. To protect the cash flow from the fluctuations, incorporating uncertainties into mine planning can be an effective approach (Leite and Dimitrakopoulos, 2007). Primarily, two types of risks are considered in the problem: economical and geological. Economical uncertainties are related to prediction of commodities' prices. On the other hand, geological uncertainties mainly include uncertainty in estimation of grade and ore tonnage. Controlling these types of uncertainty could decrease the risk of mining projects.

In this paper, an approach is proposed for long-term open pit mine production planning in the presence of uncertainties. The main source of uncertainty is the uncertainty in the estimation of grade of elements. Therefore, how to sequence the extraction of mining blocks inside an open pit mine to maximize the cash flow while minimizing the risk and cash flow fluctuation is the main problem.

The rest of this paper is organized as follows: section 2 reviews the literature regarding the problem of long-term planning with uncertainty issues. In section 3, objectives of research are briefly presented. Also, the theoretical framework of proposed research is reflected in section 4. This section includes the mathematical model and the theoretical framework. A case study is considered in section 5 and how the proposed framework is applied is highlighted in this section. Results are discussed with tables and figures to show the practicality of the proposed framework better. Finally, the future work that could be performed related the proposed framework is explained in section 6.

2. Literature review

Mine production scheduling in the presence of uncertainty has been the focus of attention from researchers and practitioners in recent years. Leite and Dimitrakopoulos (2007) provide a framework for life of mine (LOM) production scheduling based on simulated annealing and stochastic orebody representations. The authors take into account geological uncertainty on the tonnage of ore deposit and grade in their proposed mine production scheduling framework. Their suggested mine production scheduling algorithm produces a final schedule in the presence of geological uncertainties to minimize the risk of not meeting production targets and maximize the net present value (NPV). This algorithm includes the following steps:

- Designing the pit limit and mining rates: to define pit limits, conventional methods such as Lerchs-Grossman algorithm are used. Also, mining rates are defined by Milawa scheduler or by geometric constraints and mill demands. Mining rates and pit limit set in this step will be used in the next steps.
- Generating a number of schedules respecting to pit limit: a set of schedules is generated regarding the pit limit and production targets (mining rates) by conventional schedulers and simulated ore bodies. These schedules are produced based on distinct equally probable spatial distribution of grade of ore body within the deposit while honoring different constraints such as mining precedence and production target. The produced set of schedules is used for forming a cumulative distribution indicating the probability that a typical block should be mined within the time horizon (mine's life).
- Selecting an optimal schedule by implementing simulated annealing: simulated annealing approach is used to combine different schedules from step 2 to get an optimal schedule regarding the corresponding objectives. The objective of applied simulated annealing is the minimization of deviation from the production target during the life of mine over all available representations of the deposit.

The algorithm was implemented on a disseminated copper deposit. The final schedule obtained by the framework shows 26% improvement in the NPV and low chance of significant deviations from the production targets comparing to the conventional models of mine production scheduling. The proposed algorithm considers predefined pit limit, pushback, and cut-off grades. Thus, involvement of dynamic cut-off grade and uncertainties in the pit limit and pushback design would be of high value.

Halatchev and Lever (2005) propose a risk model for long-term production planning of an open pit Gold mining. Most of works performed on the uncertainty and risk's factors of mining project have just taken into consideration the geological and price uncertainty. However, they consider four types of risk as economic, technological, technical, and geological parameters. The risk model is based on the cash flow of mining project through the mine's life and tries to formulate the effect of different risk factors on the cash flow by implementing the Monte Carlo simulation approach. The focus of the proposed risk model is on two well-known indicators: payback period and NPV. The proposed risk model minimizes the risk of deviation of cash flow from the planned cash flow.

Dimitrakopoulos and Ramazan (2008) work on providing a stochastic integer programming for optimization of long-term mine production schedule. The authors propose a stochastic integer programming model considering grade and quality of ore uncertainties. The authors review three main stochastic integer programming algorithms (anticipative models, adaptive models, and anticipation/adaptive-resource models). Then, authors explained their proposed model to maximize the NPV and minimize the mining project risk profile. In their model, a number of equal probable representations of ore body are considered and the model aims to minimize the deviation from the production targets. It is noteworthy that the proposed model is primarily on the basis of anticipation/adaptive-resource models. Dimitrakopoulos and Ramazan (2008) implemented their model on a copper-gold deposit. The results show better NPV and less deviation from the target production. The model does not take into account other sources of uncertainties such as mining equipment availability. Also, one of the main assumptions in their model is that pit limit is known. Integration of pit limit design and mine scheduling would be interesting.

As one of the first papers on considering geological uncertainty into long-term mine planning, Dimitrakopoulos et al. (2002) present a simulation based optimization model to generate a number of realizations of geological uncertainties such as grade and tonnage. The authors firstly apply conditional Gaussian simulation to produce realizations. Then, based on each realization, scheduling of block extraction is performed to reach the maximum NPV. A distribution for NPV is obtained which with a confidence interval for value of cash back can be presented to mine managers and investors. The approach is applied on a disseminated low-grade, epithermal, quartz breccia-type gold deposit with 50 realizations. The proposed approach does not determine what could be the best schedule. Only it indicates the distribution of NPV.

Dimitrakopoulos and Abdel Sabour (2007) introduce a framework to show the advantages of real option valuation (ROV) comparing to NPV. The authors propose a simulation-oriented real option valuation model to consider three types of uncertainties: geological uncertainty, foreign exchange uncertainty, and variability of real options (price uncertainty). The authors use geometric Brownian motion (GBM) method (Dixit and Pindyck, 1994) and mean-reverting process (MRP) proposed by Schwartz (1997) to model the variability of market and economic variables like foreign exchange and prices. To model the grade uncertainty, the authors apply sequential Gaussian simulation (SGS) method (Dimitrakopoulos and Luo, 2004). The authors use historical data and regression to estimate the parameters of the above models. A number of equal likely realizations of grade distribution by using above models are produced. The optimization process is performed on each of the realizations. In each period, managers can decide if to close the mining project or to continue. A switch cost is applied when the project is shut down. Thus, optimization is done by taking into account this switch cost and other operating cost. Dimitrakopoulos and Abdel Sabour (2007) apply the simulation-based ROV and NPV method on an Australian gold mine with 3 years. 12 mine designs are considered as the realizations. The mining valuations obtained by these methods are the expected values (average values got from applying the methods on the 12 mine designs). It should be noted, as the major element is a precious metal, GBM method is applied for model of market variables. The results of comparisons show the high variability (variance) in valuation by NPV than the proposed ROV method. Also, cash flow of simulation based ROV through 3 years is much higher than NPV. Thus, implementing simulation based ROV can cause better investment return than traditional methods like NPV. The main reason is that in simulation real option valuation some uncertainties are modeled and absorbed. However, the authors point out that the overcome of ROV on NPV is just by one case study and the generalization requires implementation of models on more case studies. It seems that the optimization process is performed without considering cut-off grade role. In fact, the proposed optimization model presented in the paper does not involve decision making on the values of cut-off. The values of cut-off influence on the economic block values. In other words, as in each realization price pattern of gold is known, consideration of cut-off variable could be of value to adjust the schedule.

Zhang et al. (2007) present a reactive approach for mining project assessment. The proposed reactive model attempts to model the decision making about the mining schedule in each period when moving forward in time, while incorporating new information. The authors first introduce an

MILP model which seeks to maximize the discounted cash flow while optimizing the cut-off grade. Then, they model the price uncertainty by use of log-normal mean reverting model to generate price patterns through time. As time passes by, the pattern of price is updated by use of log-normal mean reverting model in each period. In the reactive model, the model calculates the expected value of NPV. Also, a model of optimization based on the conditional expectation of future price is presented. This conditional expectation does not update the price pattern. In fact, it uses only a static pattern based on the historical data which are on-hand at the first period. In the end, the authors present a model based on perfect knowledge of future price. The authors show that the NPV obtained by perfect knowledge is more than the NPV by reactive method and conditional model. The reason is clear because in the perfect knowledge model the future price is completely known. Thus, the scheduling is carried out based on certain information while in the reactive and conditional based model, the pattern of price are estimated. The authors implement three models on a case study with 16000 blocks with 25 realizations (patterns). The results show the superiority of reactive approach proposed by in the paper on conditional-based model of future price.

Ramazan and Dimitrakopoulos (2007) introduce a stochastic integer programming formulation to tackle production scheduling problem with the ore body uncertainty. The mathematical formulation aims at maximizing the NPV while minimizing the deviations from the production target (in ore tones, grade, and quality). The authors aggregated these goals as an objective function to maximize the net profit in term of the total expected NPV minus the cost of missing the production targets. To impose the grade uncertainty, a number of equally probable realizations of ore body are generated. The authors implement their formulation on a synthetic two-dimensional data set. To quantify the uncertainty within each schedule, three measures are used:

- Average deviations from the production targets
- Average of non-zero deviation from the production targets
- Probability to deviate from the production targets

Based on the objective function values and the three above-mentioned measures, a schedule is picked up as the best. The paper does not show how to select the best schedule. It seems that applying different realizations of geology uncertainty, a distribution for objective function is obtained. Also, as we do not know what realization is the real case, selecting based on the introduced measures appears to be impractical.

Boland et al (2008) present a multistage stochastic programming model for the problem of long-term mine production scheduling. The authors consider geology uncertainty, including rock type and grade, as the main stochastic nature of the problem. The proposed mixed integer multistage model involves multiple estimates of geology and tries to adopt the mining and processing capacities in later periods regarding the decisions made in the earlier. The authors first re-introduce their proposed deterministic MILP model in their previous work. Then, they apply stochastic approach in two steps: 1) scenario-dependent processing decision and 2) scenario dependent mining and processing decisions, where scenarios are different realizations of uncertainty.

3. Objectives of research

As mentioned, the problem, which is considered in this research is the long-term planning of open pit mines with accounting for grade uncertainties. Grade uncertainty is one of the main sources of not meeting the production targets in mining operations. In this paper, a long-term mine production scheduling formulation in the presence of grade uncertainty is proposed. Thus, in addition to a practical production schedule, cut-off grade values are decided by the model. The following steps are completed throughout the paper:

- Grade uncertainty modeling: to model the uncertainty in estimation of grade distribution, grade values of each block is considered to have Normal distribution with mean and standard deviation obtained by sequential Gaussian simulation method.
- Calculation of economic block value (EBV) regarding the considered grade distribution.
- Mathematical modeling with involving cut-off grade value optimization to handle both cut-off and scheduling to obtain the maximum value of expected profit.

4. Theoretical framework

Following is the proposed theoretical framework:

- Modeling and generating a number of grade distribution realizations.
- Synchronization of EBV estimation and each realization of grade distribution.
- Presenting a mathematical formulation for the corresponding problem.
- Solving the mathematical formulation with each realization of grade distribution.
- Analysis of schedules obtained in step 4 and distribution of objective function (NPV).

The first three of above items are elaborated in the current section. Other two items are explained with the case study in section 5. The proposed framework has the following improvements over the previous research:

- Consideration of both schedule and cut-off optimization.
- Estimation of EBV related to each realization of grade distribution. Since, EBV primarily depends on the value of grade of elements, each realization of grade distribution leads to a different EBV. Thus, estimation of EBV itself is a problem that should be performed with long-term scheduling.
- Producing a number of schedules which can be used for estimation of NPV distribution. NPV distribution can be used as an indicator for investors. Investors can expect profit with their desired confidence by seeing the NPV distribution.

4.1. Grade distribution modeling

To take into account the grade uncertainty, a number of simulation realizations are generated; each of these realizations represents a certain distributions for grade of elements. In this research framework, SGS is implemented to produce realizations. Based on SGS, in each realization, a value for grade of elements is assigned which obeys from the Gaussian distribution with a mean and a standard deviation. In this research, a case study is used in that grade of elements is reproduced by mean and standard deviation obtained by kriging (Journel and Huijbregts, 1978).

4.2. Synchronization of EBV and grade distribution

Since EBV depends on the value of grade of elements, determination of EBV related to each realization of grade is a problem. In other words, EBV corresponding to each realization of grade distribution has to be calculated. In this paper, to estimate the values of EBV related to each realization, an artificial intelligence method called support vector machine (SVM) (Schuurman, 2010) is applied. In this method, three features are considered for estimation of EBV. These features are as follows:

- Ore tonnage
- Mining cost
- Grade

Above features directly influence the block economic value. The values of these features in each block are used to estimate the best value of EBV for the corresponding block in each realization.

4.3. Mathematical formulation

A mixed integer programming formulation is presented to model the multiple-period extraction of blocks in the long-term. The proposed model, presented by Zhang et al. (2007) model.

$$Max\ Z = \sum_{i=1}^{N}\sum_{j=1}^{cg}\sum_{t=1}^{T} EBV_{i,j,t} * u_{i,j,t} / (1+df)^t \tag{1}$$

Subject to:

$$\sum_i \sum_j R_i * u_{i,j,t} <= MU_t \qquad \forall t = 1...T \tag{2}$$

$$\sum_i \sum_j R_i * u_{i,j,t} <= MU_t \qquad \forall t = 1...T \tag{3}$$

$$\sum_i \sum_j O_{i,j} * u_{i,j,t} <= PU_t \qquad \forall t = 1...T \tag{4}$$

$$N_{PB(i)} \cdot b_{i,t} \le \sum_{\tau=1}^{t} \sum_{k\in PB(i)} \sum_{j}^{cg} u_{k,j,\tau},\ \forall i = 1...N, \forall t = 1...T \tag{5}$$

$$\sum_j u_{i,j,t} \le b_{i,t},\ \forall i = 1...N, \forall t = 1...T \tag{6}$$

$$\sum_j c_{j,t} = 1, \quad \forall t = 1...T \tag{7}$$

$$u_{i,j,t} \le c_{j,t},\ \forall i = 1...N, \forall t = 1...T, \forall j = 1...cg \tag{8}$$

$$u_{i,j,t} >= 0 \quad b_{i,t} = 0/1, \forall i = 1...N, \forall t = 1...T \tag{9}$$

Where

t : period of scheduling (t=1,…,T)

N : number of blocks

MU^t : upper bound of mining capacity used in period t

PU_i^t : upper bound of plant capacity i in period t

$O_{i,j}$: mineral zone tonnage of block I, if it is extracted by cut-off grade j

R_i : rock tonnage of block i

cg: all of cut-off grade values

df: discounting factor

$PB(i)$: set of precedent blocks of block i

$N_{PB(i)}$: number of blocks in set PB (i)

$u_{i,j,t}$: fraction of block i extracted in period t with cut-off grade j

$b_{i,t}$: binary variable, if block i is extracted in period t it gets value of 1 otherwise 0

$c_{j,t}$: binary variable, if cut-off grade j is selected in period t it gets value of 1 otherwise 0

Eq. (1) indicates the maximization objective function. This function is the summation of discounted revenue by extraction of blocks with different cut-off grade values (maximization of profit). Eq. (2) indicates that in each period, total amount of extraction has to be less than the maximum capacity for mining. Also, Eq. (3) conveys the same concept of Eq. (2) for the processing capacity. Eq. (4) implies the precedence or slope constraint in the extraction process. Based on this constraint, the extraction of each block can be started when all its above blocks have been mined completely. Eq. (5) defines the meaning of binary variable $b_{i,t}$. Eq. (6) indicates that in each period only one cut-off grade can be imposed. Eq. (7) states the definition of binary variable for cut-off grade choosing in each period. Based on this constraint, whenever a cut-off grade value is chosen for a period, no fraction of blocks can be extracted by that specific cut-off grade in that period. At last, Eq. (8) indicates the sign constraint.

5. Case study and discussion of results

To show the practicality of the proposed framework, model #2 of Askari-Nasab and Awuah-Offei (2009) is used as the mathematical model. The mathematical formulation is run in TOMLAB/CPLEX environment (Holmstrom, 2009) with realizations of grade distributions. In this section, an illustrative example of long-term planning of Gol-E-Gohar iron ore mine in south of Iran is presented to validate the proposed framework. The main element of interest in the deposit is Iron (Fe). The process employs magnetic separators; therefore, the main criterion in selecting ore to be sent to the concentrator is the magnetic weight recovery (percent MWT) of iron ore. The contaminants present are phosphor and sulfur that are considered as secondary elements to be controlled. The open pit has 20 benches which only blocks of benches 14, 15, 16, and 17 with 3089 blocks are used for the purpose of long-term planning over 10 years. Blocks are clustered into 150 mining-cuts using fuzzy C-means method. For more information on clustering algorithms for block aggregation see Askari-Nasab (2010).

Fig. 1 shows the plan view of four benches, 14 to 17. Table 1 represents the general information of the problem. Based on this table, tonnage of rock and ore inside the four benches are around 94 and 47 million tonnes which should be extracted and processed within 10 years. The number of cuts in the benches 14 to 17 are 45, 40, 35, and 30.

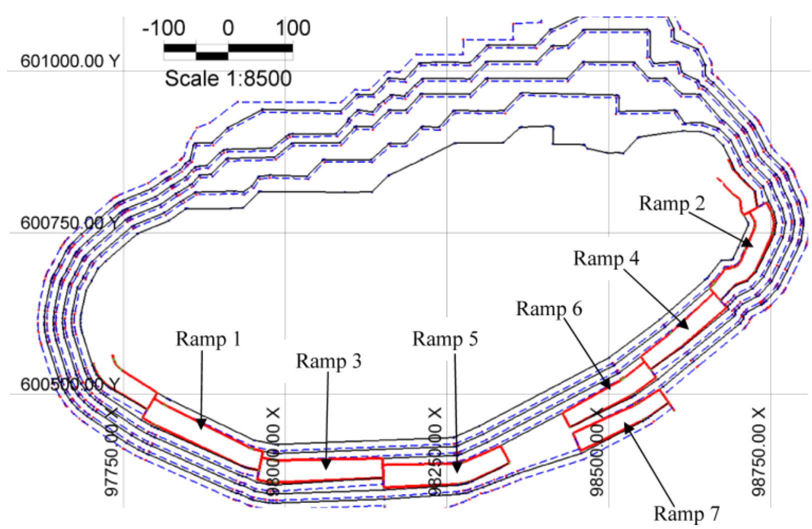

Fig. 1. Plan view of benches scheduled over a 10-year time horizon (Eivazy and Askari-Nasab, 2010)

For simplicity, only one process (process #1) is considered for processing the ore extracted from the mine. Table 2 indicates the acceptable grade range for different elements and the processing capacity. The acceptable grade ranges and processing capacity are the same over the 10 years of scheduling.

Table 1 General information of problem

	BENCH NUMBER			
	17	16	15	14
Number of blocks	614	726	820	929
Number of Cuts	30	35	40	45
Number of periods	10 years			
Total number of blocks	3089			
Total number of cuts	150			
Block size	$25 \times 25 \times 15$ (m^3)			
Total rock tonnage	94 million tonnes			
Total mineral tonnage	47 million tonnes			

Table 2 Processes' main features.

PROCESS	LOWER GRADE (%)			UPPER GRADE (%)			CAPACITY (MILLION TONNES)	
	MWT	S	P	MWT	S	P	Min	Max
Process #1	0	0	0	90	0.02	0.01	0	4.8

Fig. 2 sketches the rock code of blocks in bench 17. Mineralized zone is indicated by rock code 101. All other rock codes are waste.

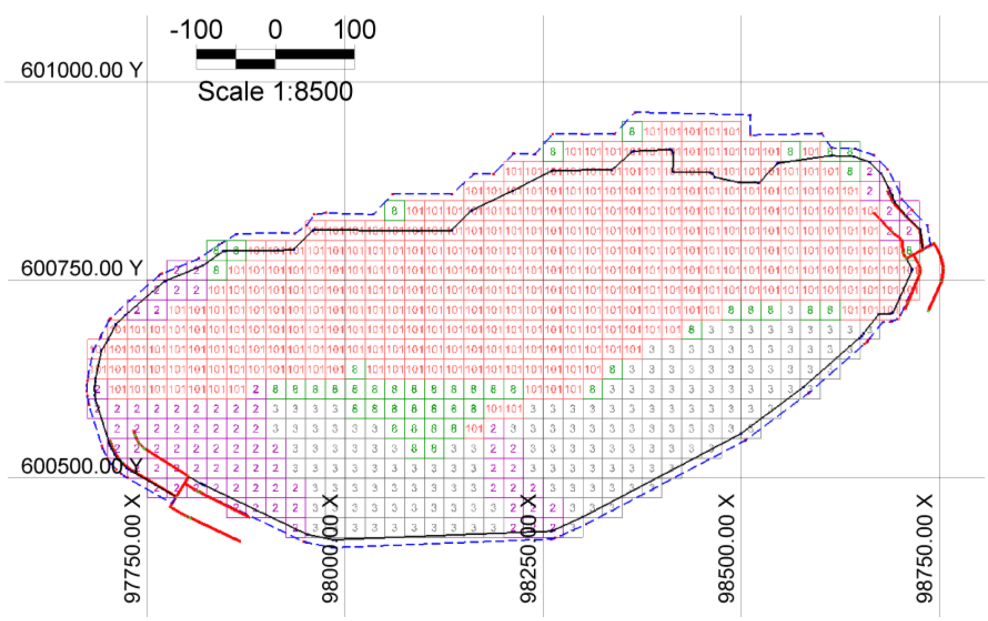

Fig. 2. Plan view of bench 17 showing waste and mineralized zone (Eivazy and Askari-Nasab, 2010)

After producing 100 realizations for grade distributions, the mathematical model is solved with each realization and the best schedule is obtained. Fig. 3, Fig. 5, Fig. 7, and Fig. 9 show the plan view of levels 14 to 17 based on the optimal production schedule with the first realization. Also, Fig. 4, Fig. 6, Fig. 8, and Fig. 10 illustrate the plan view of levels 14 to 17 based on the optimal production schedule with the realization #100. The numbers inside each block represent the year that maximum portion is extracted from that block. For instance, 10 placed in a block refers to that most of rock tonnage of block is extracted in year 10. It can be seen that each realization of grade distribution would result in an optimal schedule. Thus, with 100 realizations, we have 100 best production schedules. In each of schedules, it is determined that each block should be extracted at most in what period. For example, block # 1 can be extracted in years 8, 9, and 10 in different realizations. To aggregate the 100 optimal schedules, a rough schedule could be inferred from these 100 schedules. Fig. 11 to Fig. 14 show the rough schedule obtained by average of 100 schedules for benches 14 to 17. Each number inside each block in these figures shows the year that maximum fraction of that block is going to be extracted in that year in most of years. In other words, based on 100 schedules we know that one block at most is extracted in what periods. Thus, the most repeated period is selected as the year that block is better to be extracted.

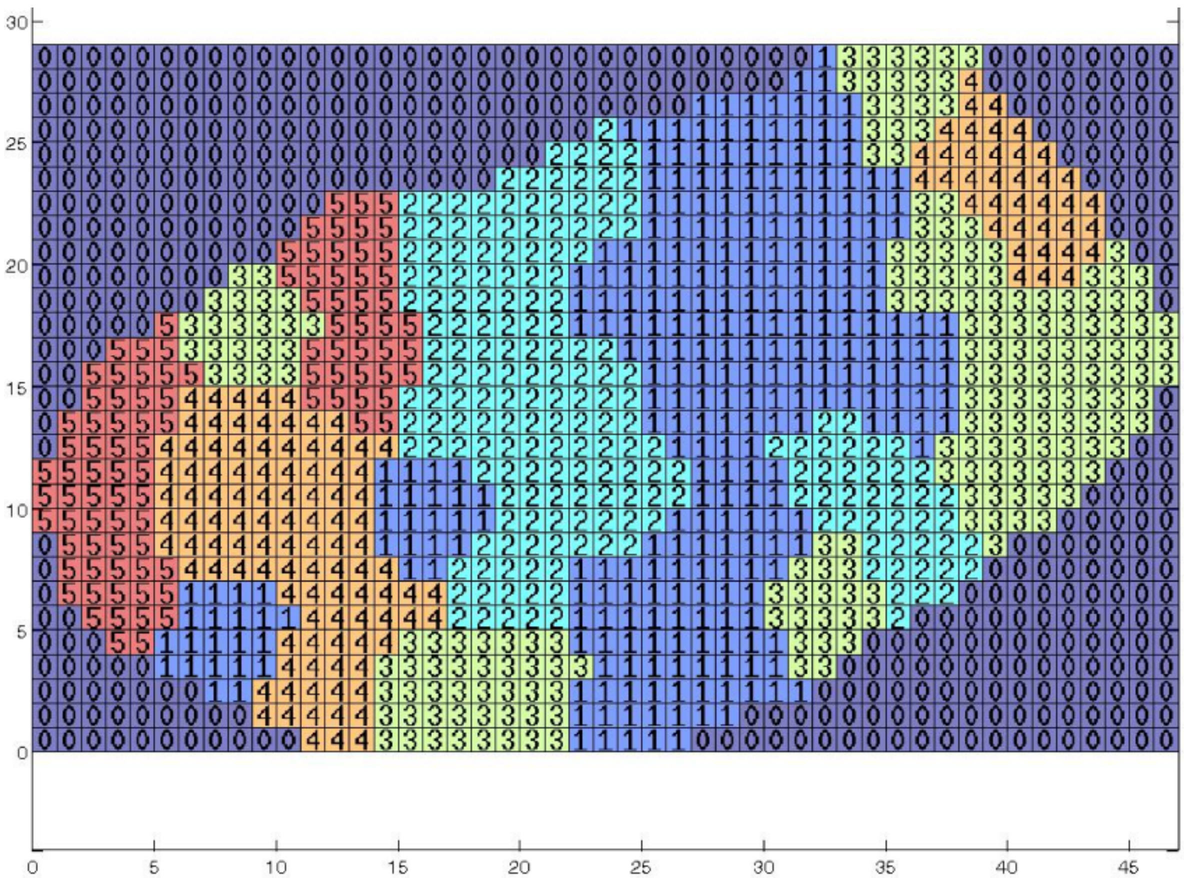

Fig. 3. Plan view f bench 14-realziation #1 (each block: 25m×25m)

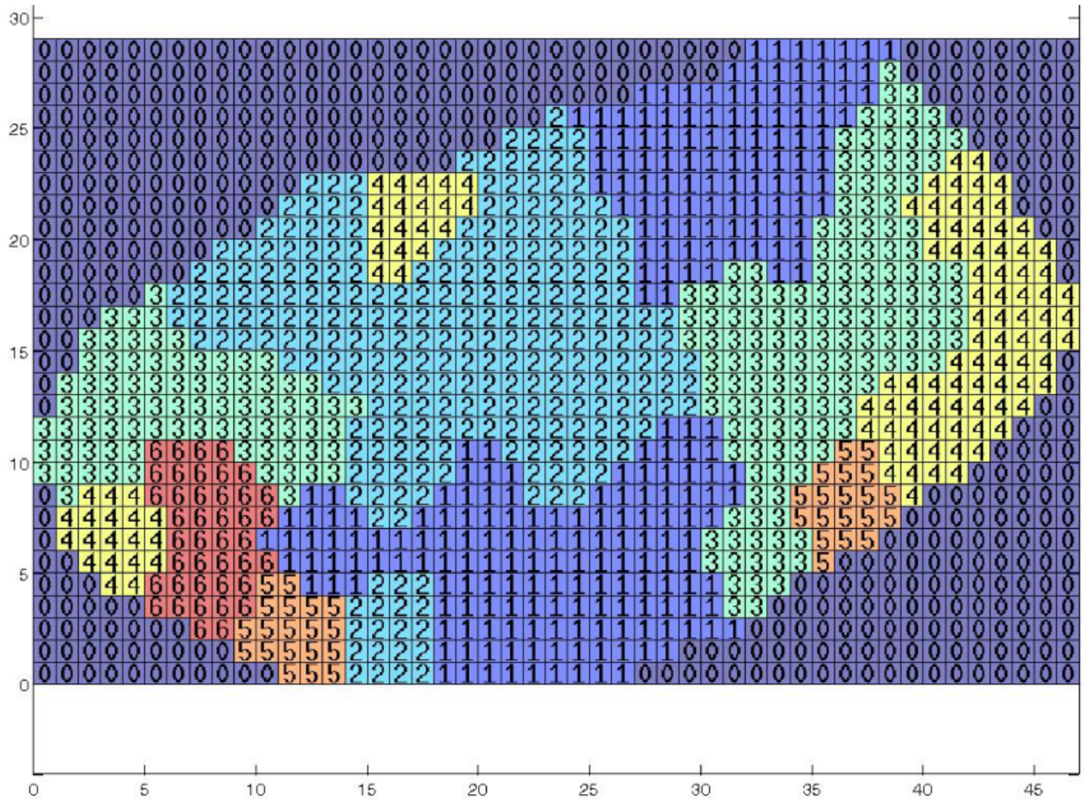

Fig. 4. Plan view of bench 14-realization #100 (each block: 25m×25m)

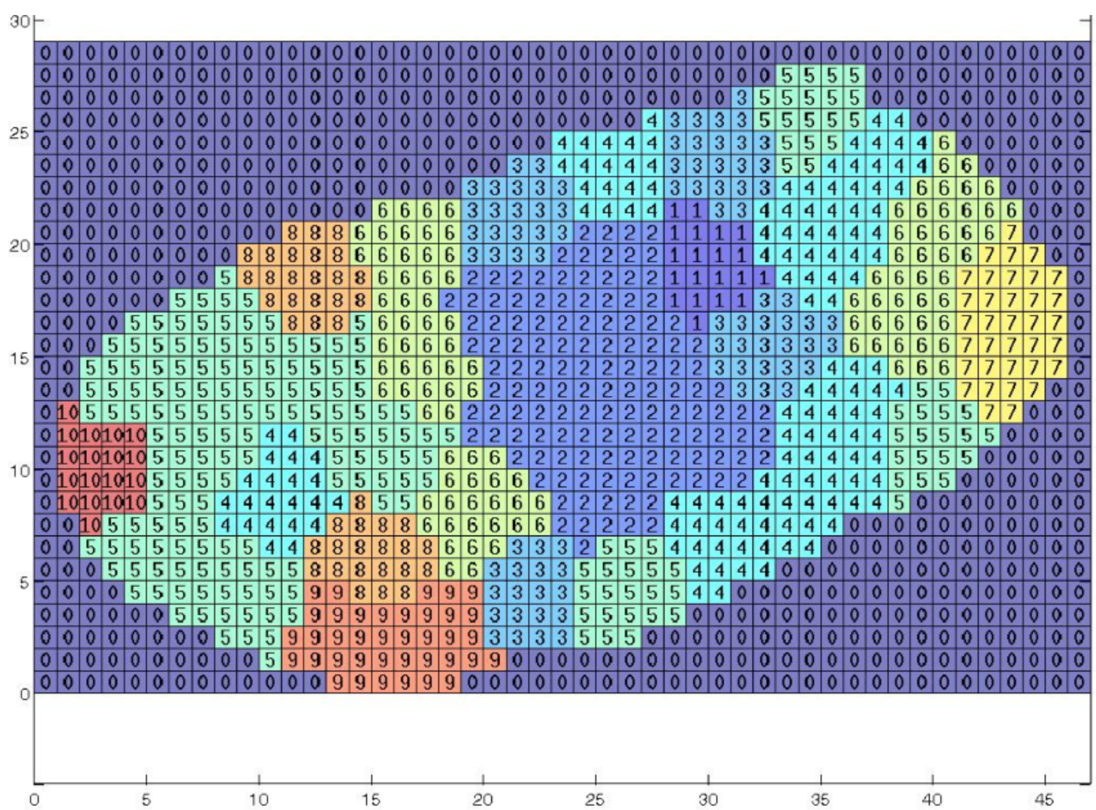

Fig. 5. Plan view of bench 15-realization #1 (each block: 25m×25m)

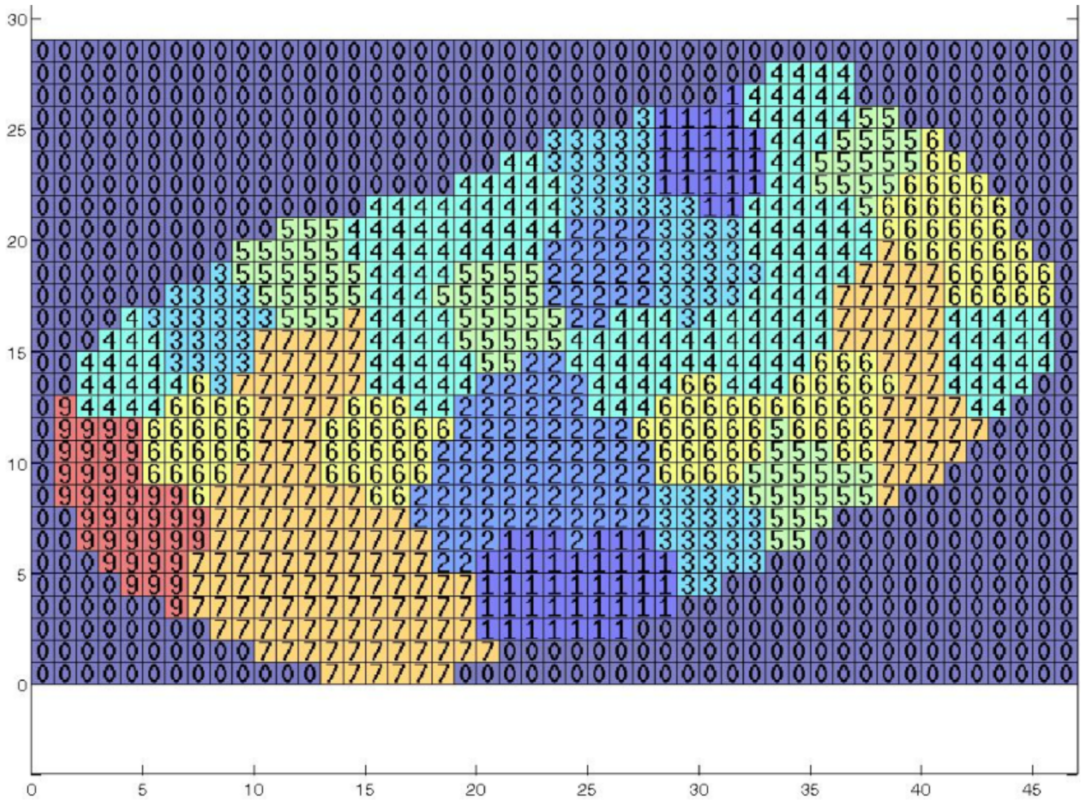

Fig. 6. Plan view of bench 15-realization #100 (each block: 25m×25m)

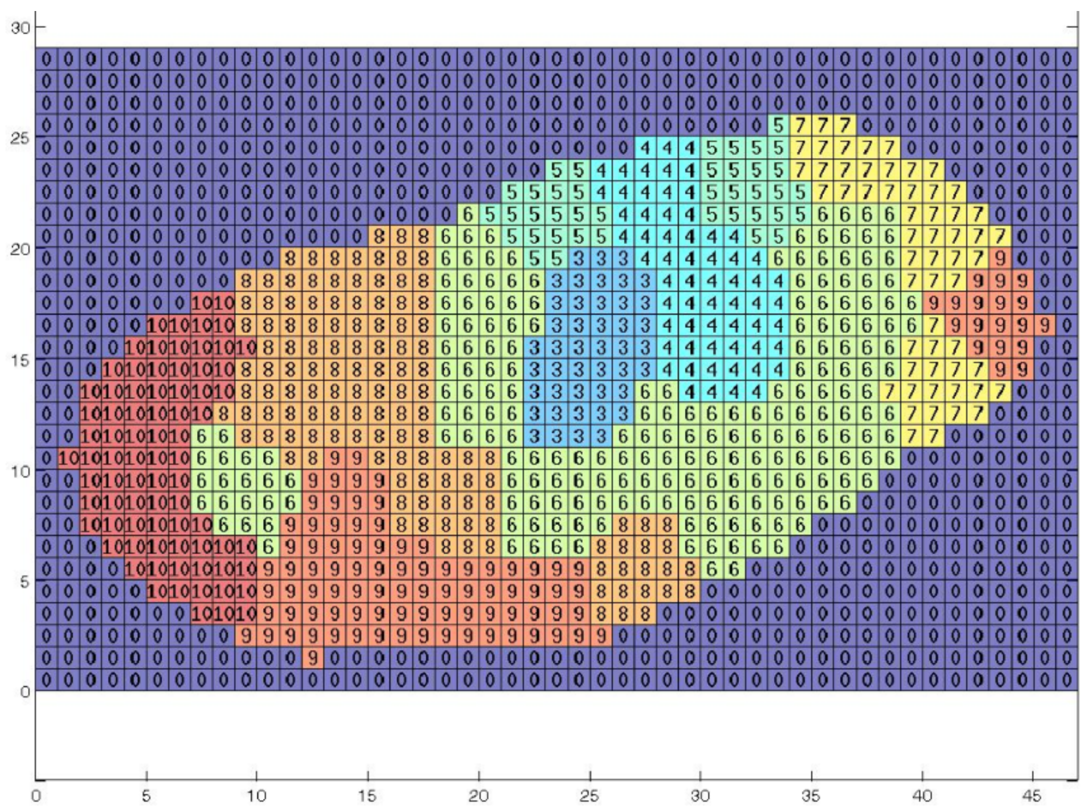

Fig. 7. Plan view of bench 16-realization #1 (each block: 25m×25m)

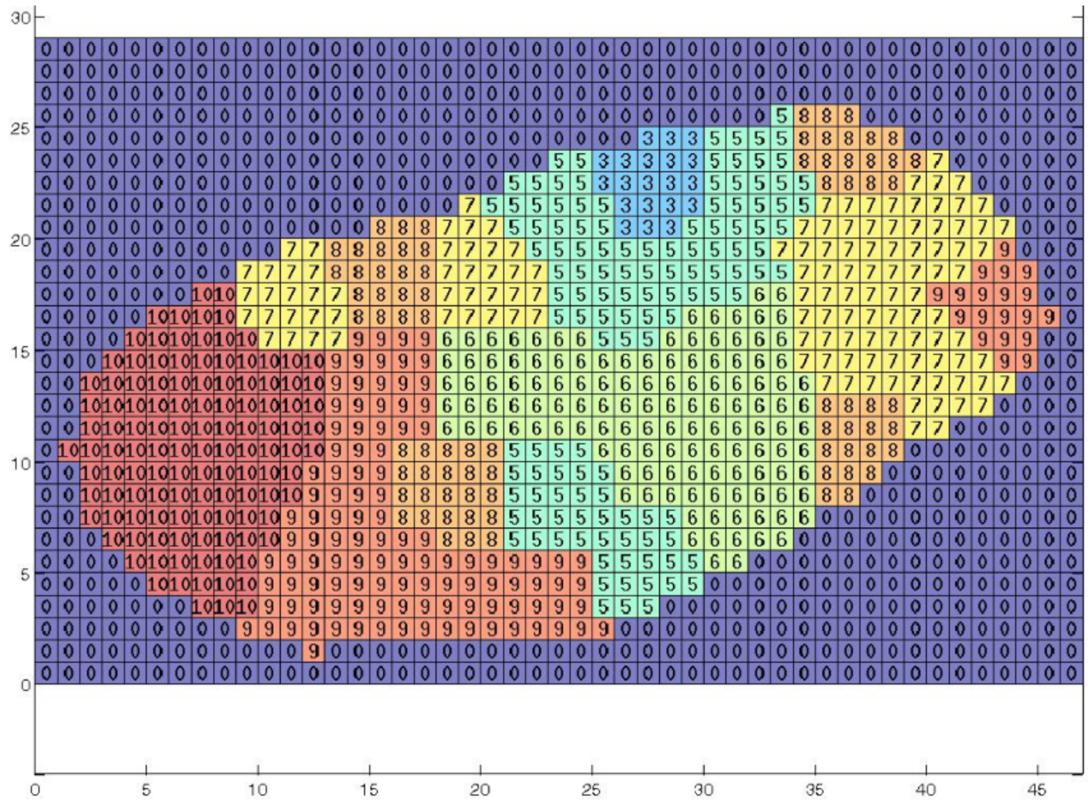

Fig. 8. Plan view of bench 16-realization #100 (each block: 25m×25m)

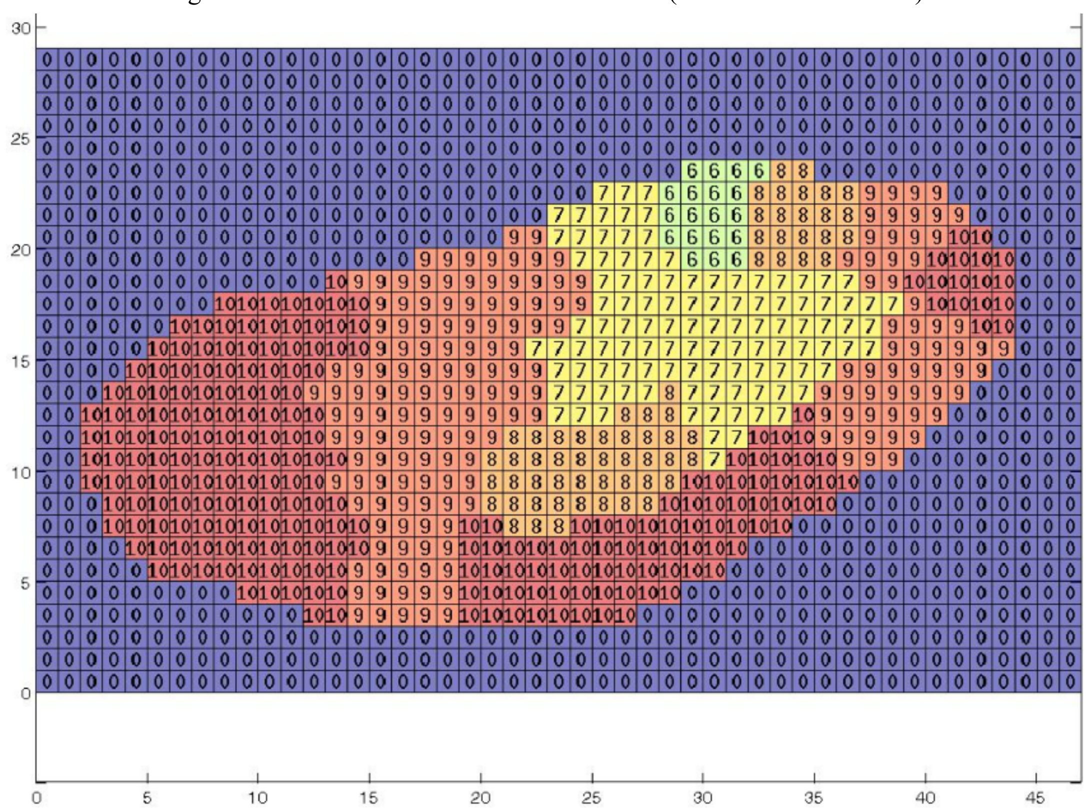

Fig. 9. Plan view of bench 17-realization #1 (each block: 25m×25m)

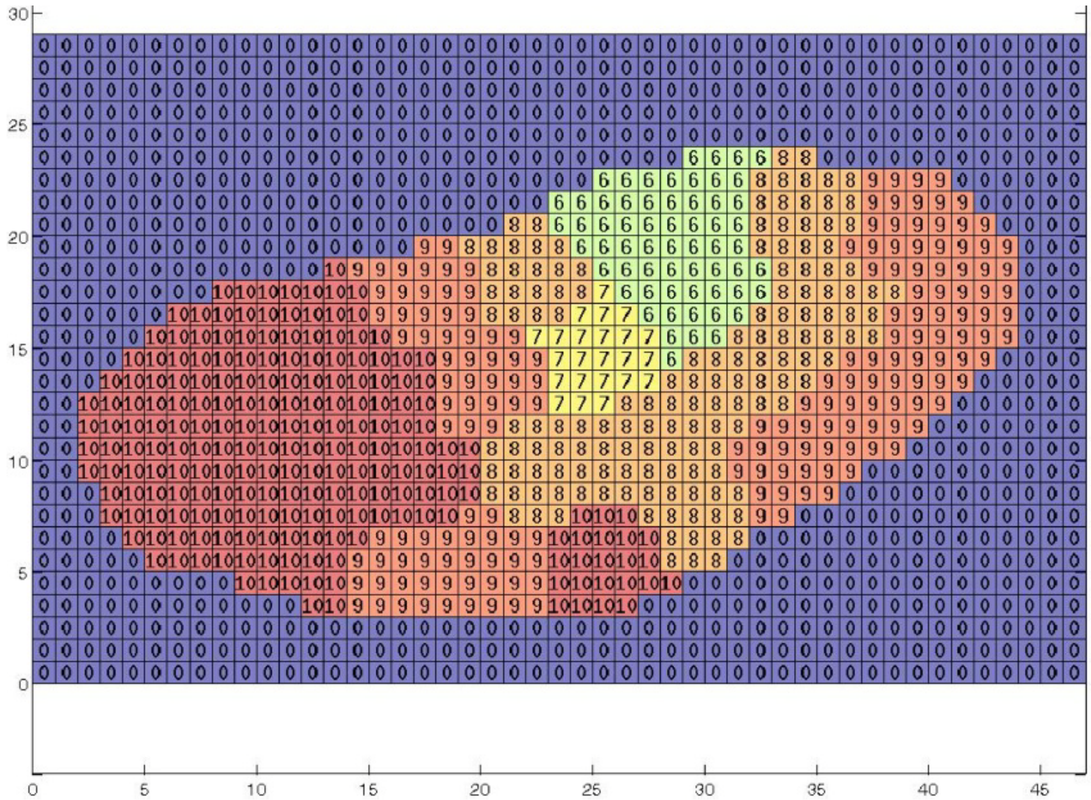

Fig. 10. Plan view of bench 16-realization #100 (each block: 25m×25m)

Fig. 15, Fig. 17, and Fig. 19 show the fluctuations of total rock tonnage mined in different 100 schedules based on 100 realizations in years 1, 8, and 9, respectively. Also, Fig. 16, Fig. 18, and Fig. 20 indicate the total ore processed in the process in different schedules in years 1, 8, and 9. As can be seen, fluctuation of total extracted rock tonnage is higher than total extracted ore in these years. Fig. 21 shows the trend of profit (objective function-net present value) in 100 realizations' schedules. Also, Fig. 22 presents the histogram plot of profit. Based on this plot, mean and standard deviation of profit are 21,648 and 6.7523 million dollars. Thus, no matter of what the grade distribution is, it is expected to earn 21,648 million dollars.

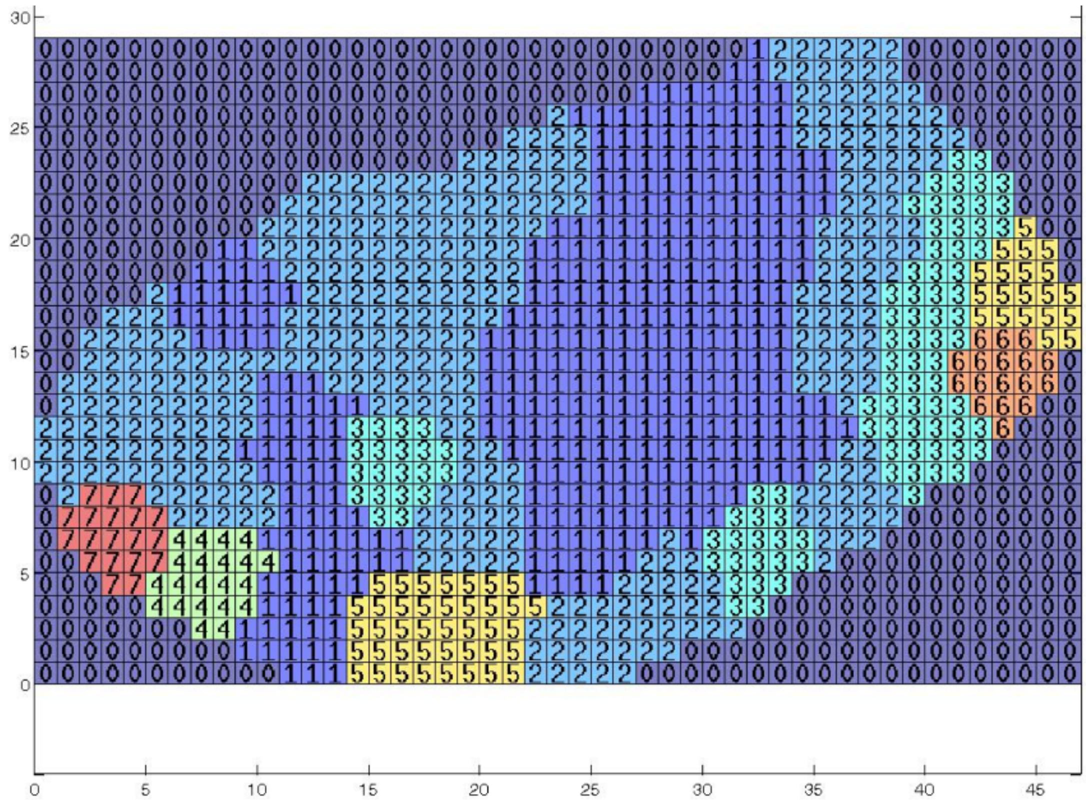

Fig. 11. Bench 14-average of realizations (each block: 25m×25m)

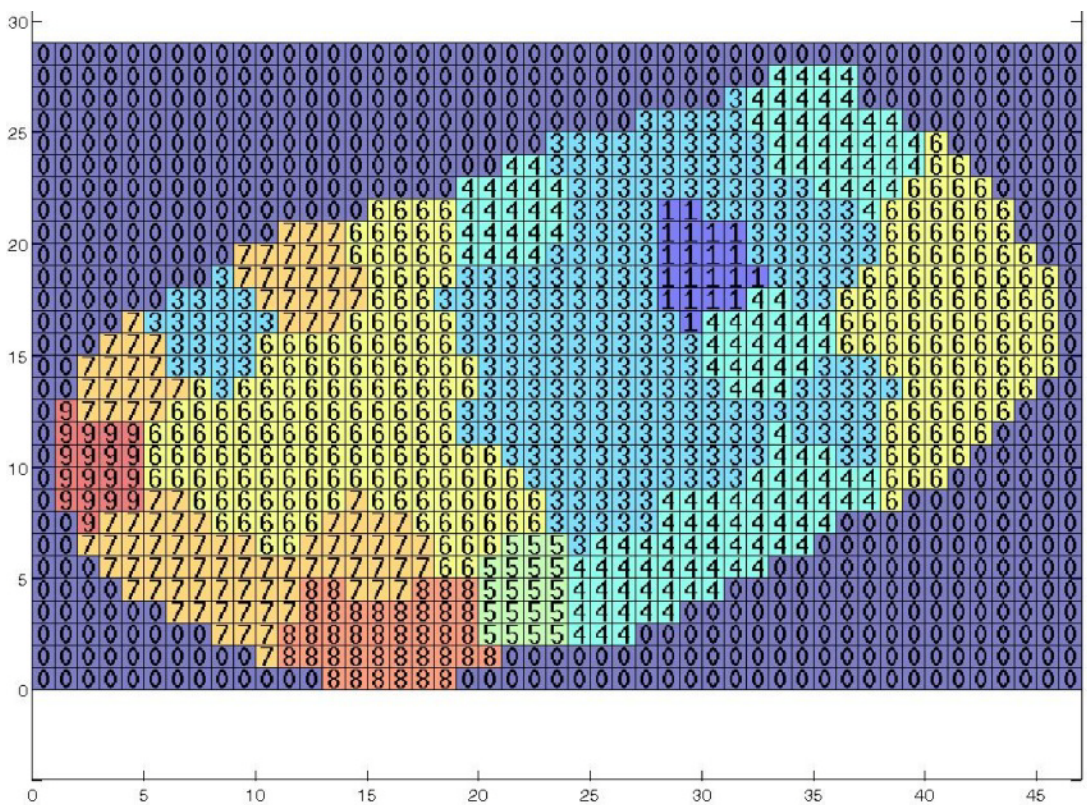

Fig. 12. Bench 15-average of realizations (each block: 25m×25m)

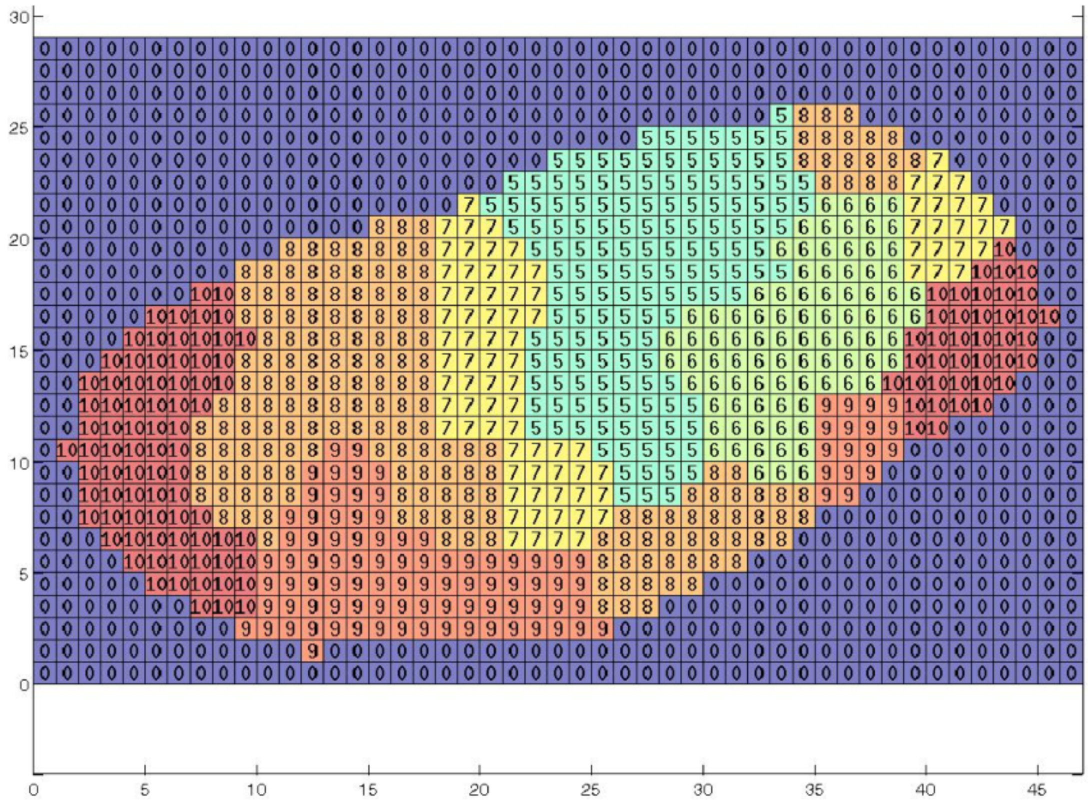

Fig. 13. Bench 16-average of realizations (each block: 25m×25m)

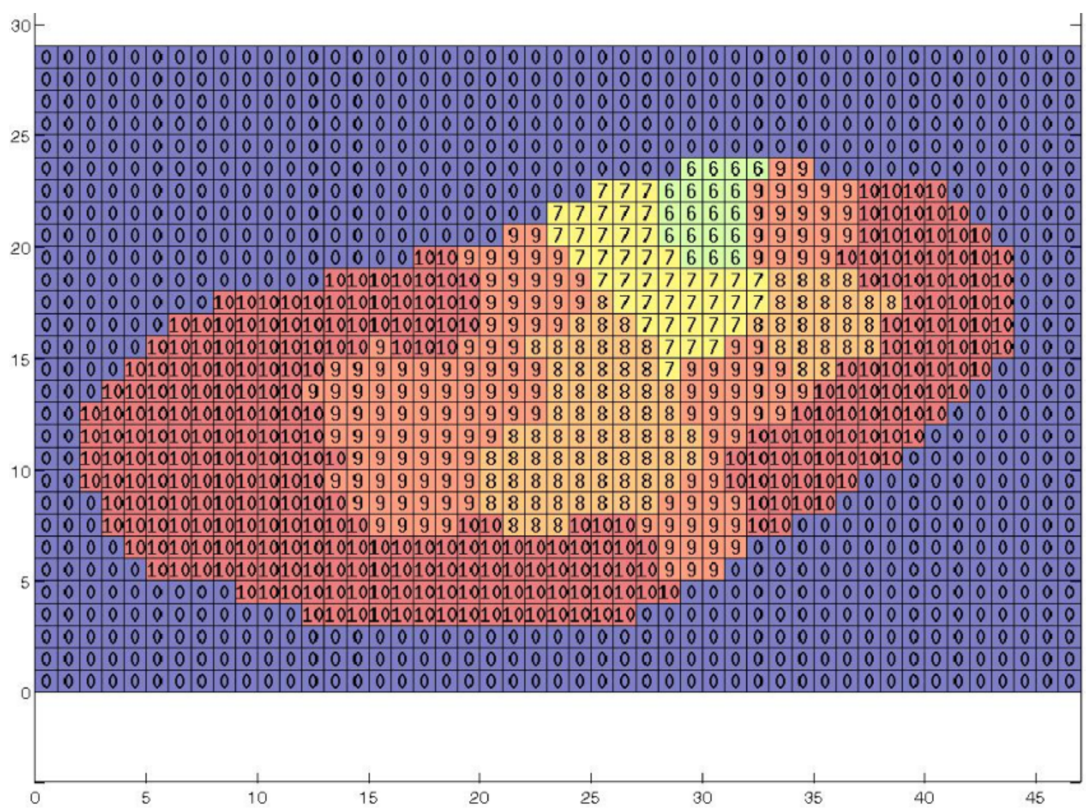

Fig. 14. Bench 17-average of realizations (each block: 25m×25m)

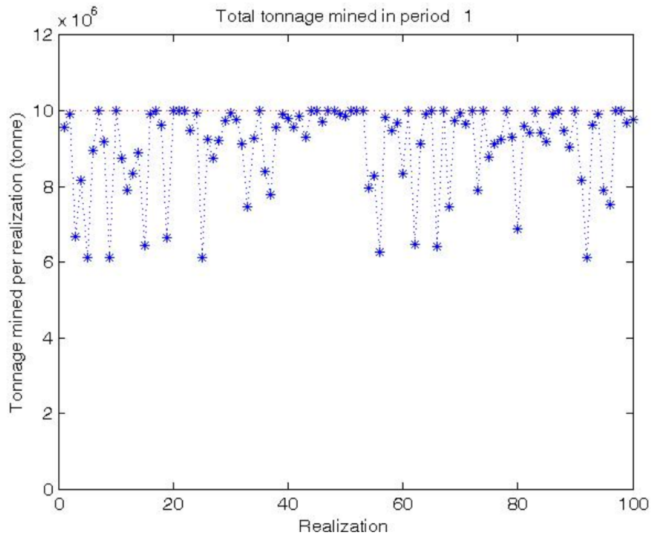

Fig. 15. Total tonnage mined in year 1 per realizations

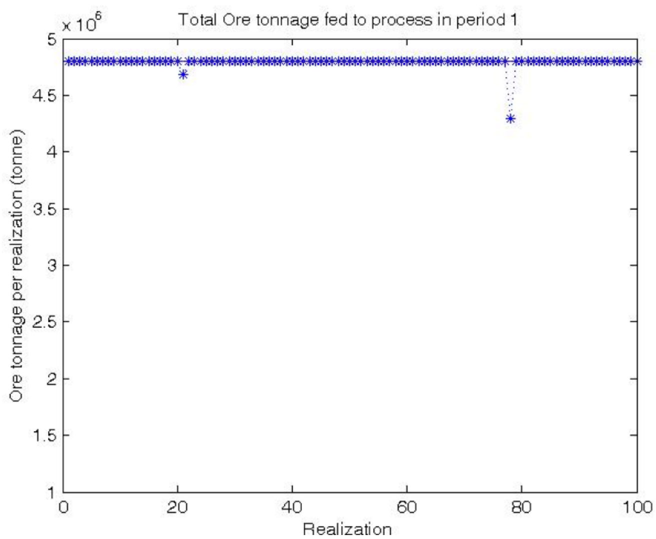

Fig. 16. Total Ore processed in year 1 per realizations

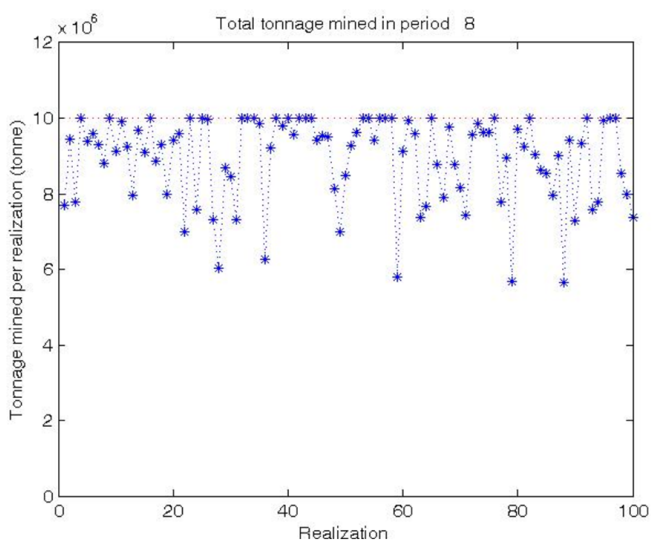

Fig. 17. Total tonnage mined in year 8 per realizations

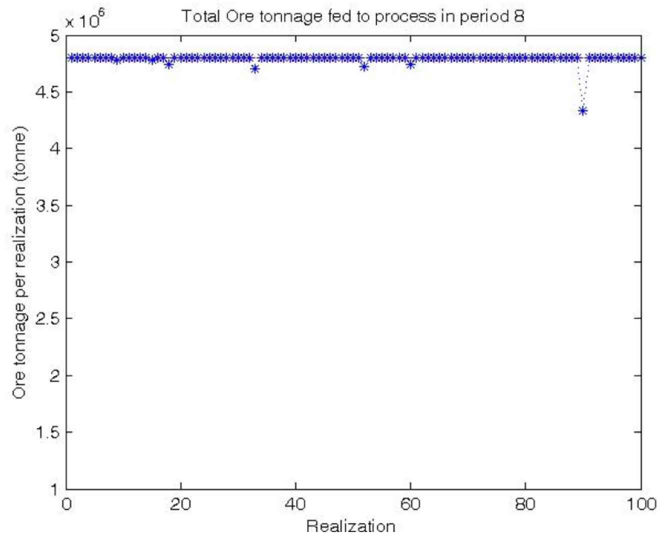

Fig. 18. Total ore processed in year 8 per realizations

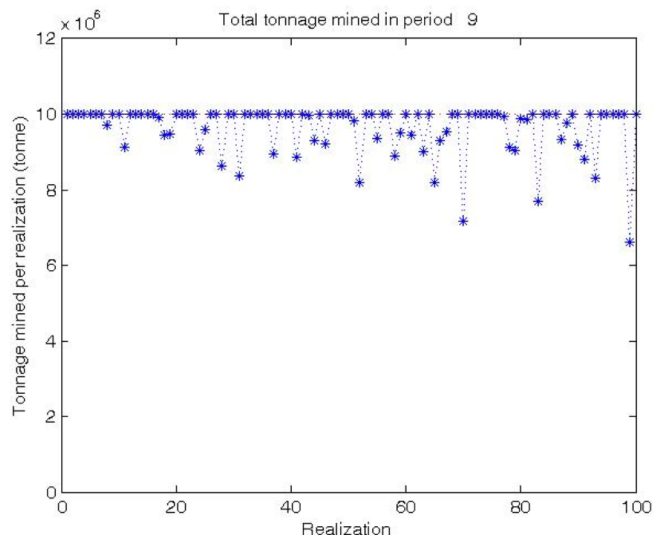

Fig. 19. Total tonnage mined in year 9 per realizations

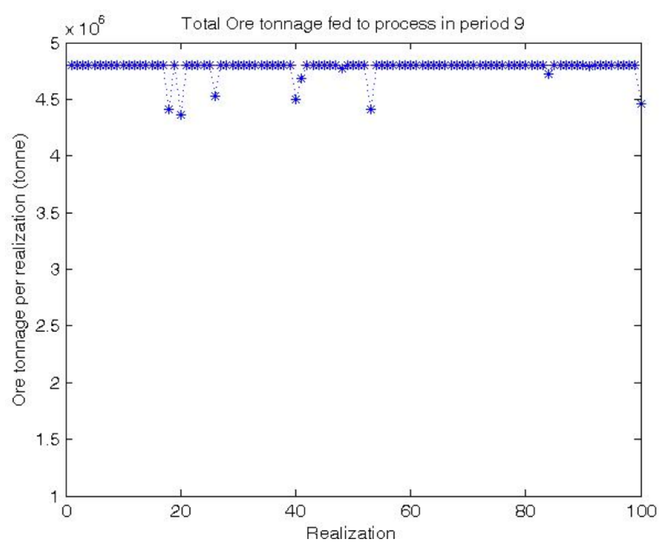

Fig. 20. Total ore processed in year 9 per realizations

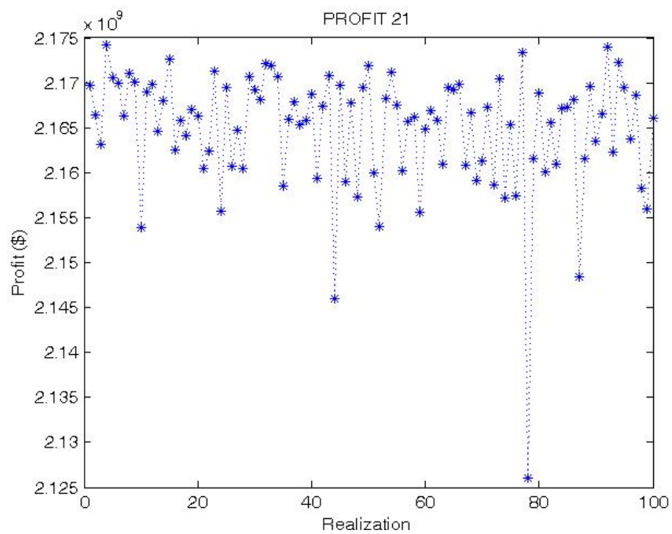

Fig. 21. Expected profit per realizations

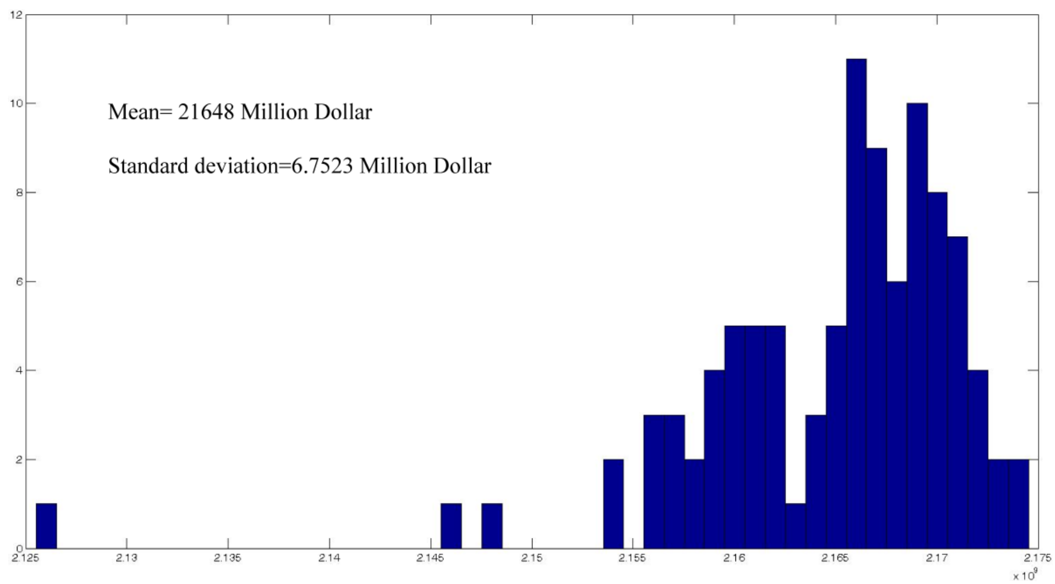

Fig. 22. Histogram plot of profit

6. Conclusions and future work

The problem of long-term mine production planning plays an important role in profitability of mining projects. Presence of uncertainty sources such as uncertainty in estimation of grade, shape of mineralized zone, and price could be potential perils for application of the long-term mine schedules. Therefore, how to involve these uncertainties in the mine scheduling stage is a challenging issue. In this paper, a mixed integer linear programming model (MILP) was developed to optimize the cash flow and cut-off grade with uncertainty approach. As the proposed model was complex, a simpler version of long-term mine production planning developed by Askari-Nasab and Awuah-Offei was implemented on a case study with generating 100 realizations of grade distribution. Hundred schedules were obtained showing how the profit changes and the period that each block most expectedly is going to be extracted. For future work, applying the proposed MILP with the grade uncertainty approach on larger models and improving the performance could be a valuable research theme. Also, integrating the price uncertainty with grade uncertainty in order to have better understanding of potential profitability of mining projects would be another noticeable work.

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