Aggregate cost minimization in hot-mix asphalt design

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Abstract

Hot-mix asphalt (HMA) is a mixture of aggregates and asphalt binder in appropriate ratios to produce a high performing material for asphalt pavements. The aggregate structure, which depends on the gradation, is an important factor in determining the volumetric properties of HMA. The design process of determining the optimal aggregate blend is currently iterative and engineers rely almost exclusively on past experience. This approach is time consuming and often results in sub-optimal HMA mixtures. This work presents linear programming (LP) optimization models, and attendant solution procedures, that minimize the HMA aggregate cost while producing high quality HMA. The models have been validated with real-life examples. The results indicate that the models can be used to replicate HMA mixes during field modifications, reduce the aggregate cost in a mixture and manage stockpile inventory. It is believed that the application of optimization models will increase the application of the Bailey method in the United States.

1. Introduction

Hot-mix asphalt (HMA) is a mixture of aggregate and asphalt binder with specific performance characteristics for pavement construction. Aggregate is the major structural (load bearing) component of HMA. Modern HMA production involves: (i) the use of different size distribution (gradation) aggregate stockpiles introduced into the plant through a set of feed bins or directly fed from the stockpiles, (ii) blending and drying in a drum dryer, and (iii) blending the hot aggregate with asphalt and storing in insulated silos for use in pavement construction. If desired, recycle asphalt product (RAP) can be introduced into the aggregate mixture after heating the aggregate to elevated temperatures. The use of quality materials (aggregates and asphalt binder), in optimal proportions, is the key to producing optimally performing HMA.

Researchers have long recognized the significance of aggregate gradation in producing high performing HMA (Richardson 1912, Goode and Lufsey 1962, Huber and Shuler 1992). The aggregate size distribution (or gradation) affects the volumetric properties such as air voids, voids in mineral aggregate (VMA), and voids filled with asphalt (VFA) of the mixture and consequently, the HMA performance. Despite this recognition, mix design methods in the United States (US) prior to the 1990s had no guidance for aggregate structure selection to achieve optimal HMA performance. The introduction of the Superior Performance Asphalt Pavement System

(SUPERPAVE) mix design standard in the US marked the introduction of specific volumetric property requirements in HMA design (Kennedy, et al. 1994). The SUPERPAVE mix design criteria for aggregate structure focuses on: (i) maximum aggregate size for the application; (ii) VMA; and (iii) aggregate skeleton. These objectives are achieved by controlling the nominal maximum particle size (NMPS) and the percent passing the 2.36 mm (US #8) and 0.075 mm (US #200) sieves (Vavrik 2000).

Even with the introduction of the SUPERPAVE mix design standards, engineers and lab technicians have largely relied on trial and error to achieve the volumetric requirements. The Bailey method is the only technique that provides a procedure to achieve the volumetric requirements of the SUPERPAVE standard (Vavrik 2000; Vavrik et al. 2002b). However, the Bailey method does not take into account the cost of the aggregate mixture. The method involves some trial and error to achieve a mixture with the desired aggregate ratios. Consequently, the first mix that meets the specified aggregate ratios is selected with no regard to the proportion of expensive or scarce aggregate stockpiles that are included in the design. This leads to sub-optimal (with regards to aggregate cost) mixes which are unacceptable in an industry where competitive bidding is the norm.

The objective of this work is to account for aggregate cost in HMA mix design through the application of optimization theory. This is done by modeling the HMA mix design problem as a linear programming (LP) optimization problem using the principles of the Bailey method. The optimization problem is then solved using LINDO API 6.0 (Lindo Systems, Inc. 2008) in MATLAB 7.7 (Mathworks, Inc. 2008). The optimization model is validated with a real life example and the practical applications of the work are illustrated with some further real-life mix design examples. The models included in this work are limited to dense-graded HMA mixes even though the principles could be applied to other HMA mixes. This work will help engineers and lab technicians design low cost HMA mixes by controlling aggregate cost in the mix. The optimisation algorithm ensures the least cost aggregate blend that meets the performance criteria is used in the mix. Also, aggregate plant managers can use the models to control stockpile inventory by including as little as possible of *scarce* stockpiles in the HMA mix. The trial and error involved in HMA mix design is significantly reduced by using the optimization modeling. Finally, this approach will increase the application of the Bailey method principles for aggregate blending during HMA mix design.

The next section of the paper covers a review of the relevant literature. Next section presents the LP models while Section 4 presents the numerical solution algorithm. The next two sections present the numerical examples used for validation and the results and discussions, respectively. Finally, Section 6 presents the conclusions followed by the list of references.

2. Relevant Literature

The objective of HMA mix design is to determine the proportions of each available component (aggregate stockpiles and asphalt binder) that will provide optimal HMA performance. The aggregates portion is the key structural component and is typically, over 94% by weight of the mix. For durable aggregate, the literature recognizes the significance of the aggregate gradation (size distribution) in producing high performing HMA (Richardson 1912, Goode and Lufsey 1962, Vavrik 2000). Different transportation authorities use different methods to design HMA mixes, for a comprehensive review of mix design methods see Vavrik (2000). Prior to the introduction of the SUPERPAVE mix design methodology in the US, the Hveem and Marshall methods were the predominant methods (Kandhal and Keohler 1985). The SUPERPAVE standards require the control of the nominal maximum particle size, voids in mineral aggregate (VMA), restricted gradation zone, and the percent passing the 2.36 mm (US #8) and 0.075 mm (US #200) sieves (Kennedy et al. 1994; McGennis et al. 1995).

The Bailey method is a systematic approach to blending aggregates to achieve the desired mixture properties (Vavrik 2000; Vavrik et al. 2002a,b). The method has been used since the early 1980s throughout the state of Illinois. The Bailey method can be used with any mix design method but the method itself is not a mix design method. The Bailey method, as discussed here, is suitable for dense-graded mixtures but can be applied to stone matrix asphalt (SMA) and fine graded mixes with some modification (Vavrik et al. 2002a,b). The Bailey method rests on two basic principles - aggregate packing, and coarse and fine aggregate definition. The degree of aggregate packing in a mixture is a function of the type and amount of compactive effort, particle shape, particle surface texture, gradation, and particle strength and durability. The Bailey method proposes an alternate definition of coarse and fine aggregate in HMA mixtures based on the mixture packing and interlock characteristics. Coarse aggregates are defined as those particles that will create voids in a unit volume and fine aggregate are the particles that fill the created voids. A particle ratio of 0.22 is used in the Bailey method to break the mixture into different fractions via control sieves (Vavrik 2000; Vavrik et al. 2002a,b). Equation (1) shows the Bailey method definition of primary, secondary and tertiary control sieves. NMPS is defined as one sieve larger than the first sieve that retains more than 10% as per SUPERPAVE terminology; and PCS, SCS and TCS are the primary, secondary and tertiary control sieves, respectively. The half sieve is defined in the Bailey method as shown in Equation (2). Using the standardized set of sieves in Table 1, and Equations (1) and (2), results in the control sieves shown in Table 2. Further to the control sieves, the method defines three aggregate ratios (Equation 3) to characterize the coarse, the coarse portion of the fine, and the fine portion of the fine aggregate in the mixture. These aggregate ratios have been shown to correlate well with the volumetric properties of the HMA mix (Vavrik et al. 2002a,b; Mohammad and Shamsi 2007). The Bailey method is based on volumetric blending of aggregate to achieve the desired aggregate ratios and hence the desired HMA volumetric properties (see Vavrik, et al. (2002a) for recommended aggregate ratios for different HMA mixes).

$$\begin{aligned} PCS &= 0.22 NMPS \\ SCS &= 0.22 PCS \\ TCS &= 0.22 SCS \end{aligned} \tag{1}$$

$$\text{Half sieve} = 0.5 NMPS \tag{1}$$

$$\text{CA ratio} = \frac{\text{\% passing half sieve - \% passing PCS}}{\text{100\% - \% passing half sieve}} \tag{2a}$$

$$\text{FA}_c = \frac{\text{\% passing SCS}}{\text{\% passing PCS}} \tag{3b}$$

$$\text{FA}_f = \frac{\text{\% passing TCS}}{\text{\% passing SCS}} \tag{3c}$$

3. Aggregate Blending Optimization

We formulated the aggregate blending problem during HMA mix design as a LP problem. The objective was to minimize the aggregate cost while producing high performing HMA that meets all the production specifications. The approach ensures the production specifications were met by taking advantage of the correlation between the specifications and the Bailey method aggregate ratios. A typical asphalt plant has a finite number of aggregate stockpiles. The aggregate blending problem is to determine the ratio of each stockpile that needs to be used in the HMA mix to produce a high performance mixture. Therefore, the percentages of each stockpile are the decision variables in this problem.

Table 1 US standard sieve sizes for HMA analysis

Sieve #	***Sieve Size***	
	US Standard	*(mm)*
1	1 1/2"	37.5
2	1"	25.0
3	3/4"	19.0
4	1/2"	12.5
5	3/8"	9.5
6	#4	4.75
7	#8	2.36
8	#16	1.18
9	#30	0.600
10	#50	0.300
11	#100	0.150
12	#200	0.075

Table 2 Bailey method control sieves

	NMPS (mm)					
	37.5	*25.0*	*19.0*	*12.5*	*9.5*	*4.75*
Half sieve	19.0	12.5	9.5	4.75	4.75	2.36
PCS	9.5	4.75	4.75	2.36	2.36	1.18
SCS	2.36	1.18	1.18	0.60	0.60	0.30
TCS	0.60	0.30	0.30	0.15	0.15	0.075

3.1. Objective Function

The objective of aggregate blending should be to minimize the cost of the aggregate used in the HMA mix. Equation (4) shows the objective function. c_j is the unit cost ($/ton) of stockpile *j*, x_j is the percentage of stockpile *j* in the mix (decision variables), and n is the number of bins/stockpiles.

$$Min \quad z = \frac{1}{100}\sum_{j=1}^{n} c_j x_j \tag{3}$$

3.2. Constraints

3.2.1 *Percentage Constraint*

This constraint is an equality constraint to ensure that the sum of the decision variables is equal to 100% (Equation 5).

$$\sum_{j=1}^{n} x_j = 100 \tag{4}$$

3.2.2 *Gradation Constraints*

Equation (6) represents the gradation constraints. g_{ij} is the percent passing sieve *i* of stockpile *j*, *m* is the number of sieve sizes included in the model, l_i is the lower gradation limit for sieve *i*, and u_i is the upper gradation limit for sieve *i*. This results in *2m* constraints which is typically equal to 24 constraints in the US (Table 1). l_i and u_i must be 0 and 100% for all sieves except on sieves that the agency has a specification for the mix.

$$l_i \le \sum_{j=1}^{n} g_{ij} x_j \le u_i \quad \text{for i = 1, 2,} \cdots \text{m} \tag{5}$$

3.2.3 *Maximum Particle Size (MPS) Constraint*

Equation (7) represents the constraint to ensure the intended HMA maximum particle size is maintained by the solution. The maximum particle size is ensured by using the nominal maximum particle size as defined earlier. This is done by changing the upper bound of the next sieve below the NMPS sieve to 90%. Equation (7) uses the assigned sieve numbers in Table 1.

$$u_{(nmps+1)} = 90 \tag{6}$$

3.2.4 *Bailey Method Aggregate Ratios*

The Bailey method applies three aggregate size ratios (CA, FA_c and FA_f ratios) to control the volumetric properties of the HMA mix (Equation 3). We used two modeling approaches to model the CA ratio constraint – modeling as a range or a specific value. Equation (8) represents the constraint, if the CA ratio is modeled as a range. a is the sieve number for the half-sieve, b is the sieve number for the PCS, CA_l and CA_u are the lower and upper bounds of the CA ratio, respectively. The half-sieve and PCS are determined from Table 2 and the given NMPS. Equation (8) results in two constraints.

$$CA_l \le \frac{\sum_{j=1}^{n}\left(g_{aj} - g_{bj}\right)x_j}{100 - \sum_{j=1}^{n} g_{aj}x_j} \le CA_u \tag{7}$$

Alternatively, the CA ratio constraint can be modeled as a specific value (Equation 9). This is useful, for instance, in cases where the engineer is trying to correct for field deviation and therefore has a specific CA ratio value for the mix.

$$\frac{\sum_{j=1}^{n}\left(g_{aj} - g_{bj}\right)x_j}{100 - \sum_{j=1}^{n} g_{aj}x_j} = CA \tag{9}$$

We used a similar approach (modeling for a range and a specific value) for the FA_c constraint modeling. Equations (10) and (11) represent the inequality and equality constraints, respectively. FA_{cl} and FA_{cu} are the lower and upper bounds of the FA_c ratio, respectively, and c is the sieve number for the SCS.

$$FA_{cl} \le \frac{\sum_{j=1}^{n} g_{cj}x_j}{\sum_{j=1}^{n} g_{bj}x_j} \le FA_{cu} \tag{10}$$

$$\frac{\sum_{j=1}^{n} g_{cj}x_j}{\sum_{j=1}^{n} g_{bj}x_j} \le FA_c \tag{11}$$

Similarly, Equations (12) and (13) represent the inequality and equality FA_f constraints, respectively. FA_{fl} and FA_{fu} are the lower and upper bounds of the FA_f ratio, respectively, d is the sieve number for the TCS and the previously defined terms apply.

$$FA_{fl} \le \frac{\sum_{j=1}^{n} g_{dj} x_j}{\sum_{j=1}^{n} g_{cj} x_j} \le FA_{fu} \qquad (12)$$

$$\frac{\sum_{j=1}^{n} g_{dj} x_j}{\sum_{j=1}^{n} g_{cj} x_j} \le FA_f \qquad (13)$$

There can be three to six Bailey method aggregate ratio constraints. However, in order not to over constrain the model, the four constraints are recommended for a problem (two equality and a tight range for the third).

3.2.5 *Lower and Upper Bound Constraints*

There are technological and regulatory reasons why an engineer will require lower and upper limits on the percentage from a particular stockpile. For instance, many transportation authorities have a maximum percentage of recycle asphalt product (RAP) that can be used in a mix. Also, a mix design that requires 2% of a particular stockpile may be difficult to achieve since it requires a very low conveyor belt speed for regular production rates. Equation (14) represents the constraint for the lower and upper bounds imposed on the solution. l_i and u_i are the lower and upper bounds on the decision variable x_i, respectively. By ensuring that all l_i are greater than or equal to zero, the non-negativity constraint of an LP problem is satisfied by the lower bound constraint. Hence the non-negativity constraint is not explicitly built into the LP model in this work.

$$\begin{aligned} & l_j \le x_j \le u_j \quad \text{for } j = 1, 2, \cdots n \\ & l_i \ge 0 \\ & u_i \le 100 \end{aligned} \qquad (14)$$

These constraints are the preferred way to manage stockpile inventory using these models. Provided the engineer can estimate the amount of stockpile *i*, that will be available in the production period, he can then convert that to the maximum percentage of stockpile *i*, that can be included in the mix. Alternatively, the engineer could arbitrarily make the unit cost of stockpile *i* high so that as little as possible is used in the mix. This alternative is not the best since it discourages the use of the particular material and may result in less than the inventory being used over the production period.

4. Numerical Solution Procedure

LINDO is designed to solve a wide range of optimization problems, including linear programs, mixed integer programs, quadratic programs, and general nonlinear non-convex programs (Lindo Systems, 2009a,b). The linear programming solvers in LINDO are designed to solve the LP problem in Equation (15). The solvers return the optimal solution, $\mathbf{x}^*$, and the optimal slack/surplus values as well as the optimal solution and slack/surplus values for the dual problem. LINDO also includes algorithms to conduct sensitivity analysis of the objective function coefficients, **c**, and the right-hand side (RHS) coefficients, **b**. There are three linear solvers in LINDO - the Primal Simplex, Dual Simplex, and the Barrier Methods (Lindo Systems, 2009b). The nature of the LP problem determines which of the three algorithms will be the most efficient. The LP problems discussed in this paper are not exceptionally complicated in themselves. We used the Dual Simplex because it tends to do well on sparse models with fewer columns than rows or models that are primal and/or dual degenerate (Lindo Systems, 2009b).

$$\begin{aligned}&\text{minimize } \mathbf{c}^{T}\mathbf{x}\\&\text{subject to } \mathbf{A}\mathbf{x} \geq \mathbf{b}\\&\mathbf{u} \geq \mathbf{x} \geq \mathbf{l}\end{aligned} \tag{15}$$

LINDO API is an interface that allows software developers to incorporate LINDO's optimization algorithms into their own application programs. It allows a person to access the LINDO solvers from the MATLAB environment through the MATLAB executable file (MEX-file), mxLINDO. In this work, we developed MATLAB routines to read the input data from Excel files and formulate the vectors and matrices that describe the mix design LP problem. We then accessed the LINDO solution algorithms by issuing calls to mxLINDO with these matrices and vectors, and other mxLINDO input. These calls to LINDO returned the optimal solutions to both the primal and dual problem and the ranges of the objective function coefficients and RHS coefficient for which the optimal basis will remain the same.

5. Practical Applications

5.1. Case Study

We used a real-life mix design problem from Washington State to illustrate the practical applications of the work as well as to verify and validate the models and solution procedures. The contractor had to design a 12.5 mm HMA mix for a Washington State Department of Transportation (WSDOT) project. The contractor submitted an aggregate blend of 74%, 16% and 10% respectively of the 1/2" × 0, 1/2" × 1/4" and RAP (Table 3). The 1/2" × 0 and 1/2" × 1/4" crushed rock were purchased at $ 7.00 and $8.50 per ton, respectively. The value of the recycled asphalt product (RAP) is difficult to estimate since this contractor will usually not sell it to a third party. Other companies are charged a fee to dump their asphalt concrete removed from highways. The company then crushes this recycled pavement into acceptable gradation for use as RAP. The asphalt in the RAP offers an additional value to using this product in the HMA mix (RAP reduces the amount of *new* asphalt necessary). Hence, the cost (to the contractor) of using RAP is the crushing cost less the revenue from receiving and the cost savings from reduced asphalt consumption during HMA production. For the purposes of this work, the value of the RAP was estimated at $2.00/ton. Consequently, the cost of the aggregate blend in the designed mix is $ 6.74/ton.

The designed mix resulted in a VMA of 13.1% (less than the recommended 14% but above the minimum 12.5%. The designed VFA was 67% which is within the specified 65-75%. The designed mix also met all the gradation specifications of the WSDOT 12.5 mm HMA mix. Using the designed aggregate blend, the CA, FA_c and FA_f ratios were calculated to be 0.260, 0.468 and 0.371, respectively. The FA_c and FA_f ratios are within the ranges recommended by Vavrik et al. (2002a) for a 12.5 mm mix (0.35 to 0.5). Vavrik et al. (2002a) recommends that the CA ratio for a 12.5 mm mix should be between 0.5 and 0.65. However, data from the contractor and other successful mixes suggests that a CA ratio between 0.25 and 0.4 gives good results for these stockpiles.

First, we set up the LP problem, based on the models presented in prior, with the stockpile data and the ratios set to the exact same ratios of the contractor's design (the FA_f ratio was set to a range in order not to over constrain the problem due to round-off errors). The problem results in a LP problem with three decision variables and 29 constraints (the percentage constraint, 24 gradation constraints and four Bailey method ratio constraints). We set the lower and upper bounds of the decision variables to zero and 100, respectively, except for the upper bound of the RAP which was set to 20% per WSDOT specifications. The purpose of this scenario was to evaluate the existence of multiple solutions to obtaining the contractor's designed mix. Also, this scenario allows one to evaluate the ability of the models to reproduce a particular blend.

Table 3 Stockpile material gradations

Sieve Size	*Stockpile Gradation (% Passing)*			*Gradation Limits (% passing)*	
(mm)	*1/2" × 0*	*1/2" × 1/4"*	*RAP*	*Min*	*Max*
37.55	100.00	100.00	100.00		
25.00	100.00	100.00	100.00		
19.00	100.00	100.00	100.00	100	100
12.50	95.93	95.98	98.20	90	100
9.50	82.38	59.76	88.20		90
4.75	54.49	3.34	71.30		
2.36	38.19	1.40	60.00	28	58
1.18	24.95	1.27	44.00		
0.60	18.12	1.21	25.50		
0.30	10.30	1.17	22.30		
0.15	6.39	1.16	10.80		
0.075	4.76	1.14	8.50	2	7

Further, we prepared another problem with the same data but by setting CA ratio to be between 0.25 and 0.4 while the FA_c and FA_f ratios were set to be between 0.35 and 0.5 each. This problem results in a problem with three decision variables and 31 constraints (the percentage constraint, 24 gradation constraints and six Bailey method ratio constraints). The lower and upper bounds of the decision variables were the same as the first problem. This will be the ideal input for an initial design if the contractor were to do the design with the approach presented here. These two problems were solved using the LINDO solution algorithms through the LINDO API 6.0. We also conducted sensitivity analysis of the optimal solution. The results are discussed in the next section.

5.2. Results & Discussions

Table 4 shows the optimal solutions, x_i^*, (mix design) of the two problems, the Bailey method ratios for the optimal mix design and the optimal value (cost/ton). We obtained the same solution as the contractor's design for the first problem. Thus, one can conclude that there is no *better* (cheaper) solution that will result in a blend with the same ratios as the contractor's design. In fact, the range of acceptable changes of unit costs (Table 7) suggests that the unit costs may be irrelevant in obtaining that solution. The solution, however, shows that one could reproduce a mix of the same properties (Bailey method ratios) using the proposed approach. The ability to reproduce a blend of the same characteristics is crucial in making field modifications to a particular mix design. For instance, if samples of the stockpile gradations show a consistent deviation, one can easily correct for the mix by obtaining a solution of the LP problem with the new gradations but the same aggregate ratios. Since the volumetric properties of the HMA mix are a function of the Bailey method ratios of the aggregate blend, the volumetric properties of such a modified mix should be close to the original.

Table 4 Optimal solution and aggregate ratios

	Mix 1	Mix 2
1/2" × 0 Ratio (%)	74	80
1/2" × 1/4" Ratio (%)	16	0
RAP Ratio (%)	10	20
CA ratio	0.260	0.363
FA_c ratio	0.468	0.461
FA_f ratio	0.371	0.371
Cost/ton ($/ton)	6.74	6.00

Mix 2 is the optimal solution to the design problem. The solution shows that it is possible to obtain a mix with acceptable volumetric properties without using any material of the most expensive stockpile (1/2" × 1/4"). This illustrates the benefit of using the proposed approach for aggregate blending during HMA mix design. Even though this results in an apparently marginal reduction ($ 0.74/ton) in the aggregate cost, this might be the difference between two contractors in an industry that is fiercely dependent on the cost at bid time. Of course, the overall reduction in cost of an asphalt paving job may be significant depending on the amount of asphalt to be used on the project. Furthermore, the environmental benefits of using more RAP is not captured in this analysis.

Table 5 shows the slack or surplus variable optimal solutions for constraints used to model the aggregate specifications. Even though the LINDO solution reports the optimal values of the slack/surplus variables for all the constraints, all except the ones reported in Table 5 were found to convey meaningful information. The percentage constraint will always have a slack of zero. The slack/surplus optimal values of the Bailey method aggregate ration constraints cannot be used to make any conclusions about the Bailey method ratios because the ratios appear in both the right-hand side (RHS) and left-hand side (LHS) of the constraints. Interestingly, Table 5 shows that none of the agency specifications are controlling constraints on the optimal solution.

Table 5 Slack/surplus values for gradation specification constraints

	Mix 1 (%)	Mix 2 (%)
12.5-mm sieve lower bound surplus	6.2	6.4
9.5-mm sieve upper bound slack	10.7	6.5
2.36-mm sieve upper bound slack	23.5	15.5
2.36-mm sieve lower bound surplus	6.5	14.5
0.075-mm sieve upper bound slack	2.4	1.5
0.075-mm sieve lower bound surplus	2.6	3.5

Table 6 and Figure 1 show the gradations of the two mixes. Both mixes respect all the gradation limits as modeled by the constraints. Mix 2 is coarser than Mix 1, with a CA ratio of 0.363 (an increase of 0.103). Also, the FA_c ratio is slightly lower than the contractor's design (Table 4). As CA ratio increases or FA_c ratio decreases, the VMA increases (Vavrik et al. 2002; Mohammad and Shamsi, 2007). Also, an increase CA ratio or a decrease in FA_c ratio will cause the VFA to increase (Mohammad and Shamsi 2007). Consequently, Mix 2 is likely to produce a mix with higher VMA and VFA than Mix 1 which had a VMA lower than the recommended 14%. The amount of material passing the 0.075 mm sieve is also slightly higher for Mix 2. Higher proportions of material of this size is known to increase the stiffness of the HMA mix. This is, however, not considered to be a significant issue given the slight increase in the material of this size.

Table 6 Gradation of optimal mixes

Sieve Size (mm)	***Gradation (% Passing)***	
(mm)	*Mix 1*	*Mix 2*
37.5	100.0	100.0
25.0	100.0	100.0
19.0	100.0	100.0
12.5	96.2	96.4
9.5	79.3	83.5
4.75	48.0	57.9
2.36	34.5	42.5
1.18	23.1	28.8
0.600	16.2	19.6
0.300	10.0	12.7
0.150	6.0	7.3
0.075	4.6	5.5

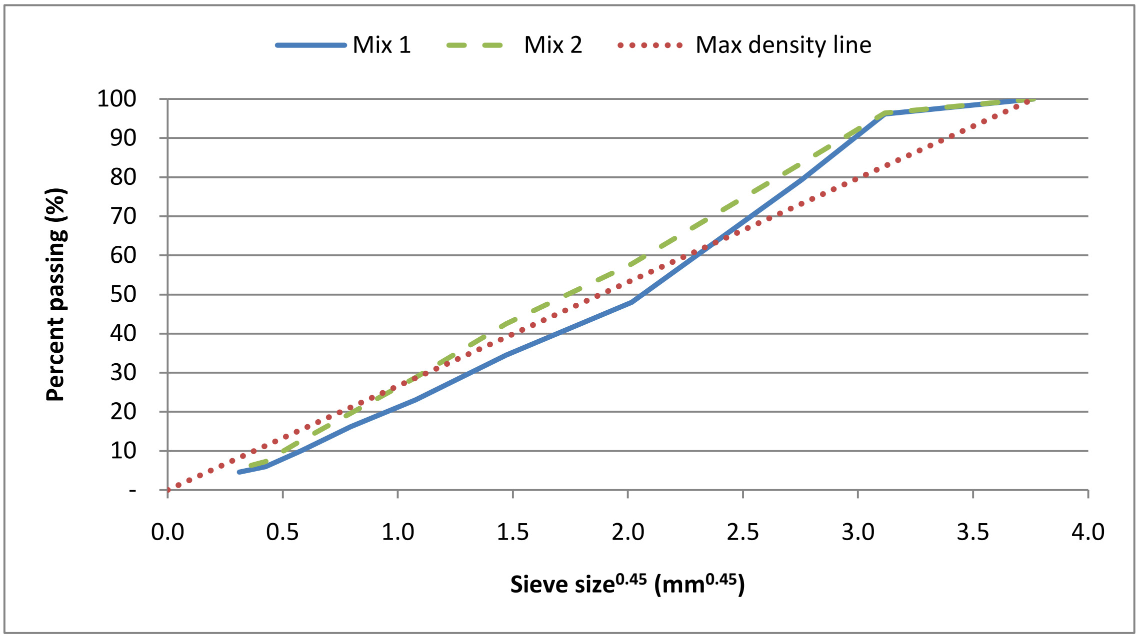

Figure 1 Gradation of optimal mixes.

It should be noted that the amount of material passing the 0.075 mm sieve is still within the WSDOT specification.

Table 7 shows the results of the sensitivity analysis on the coefficients of the objective function (unit costs of aggregate stockpiles). The ranges shown in Table 7 are the ranges for which the solution basis will remain unchanged. Sensitivity analysis of the RHS values is not reported here because it does not provide any meaningful information. The sensitivity analysis of the gradation constraint RHS values does not provide any information that cannot be inferred from the slack/surplus variables. The Bailey method ratio constraint RHS values do not provide any useful for the same reason the slack/surplus values are not useful. The sensitivity analysis shows that Mix 1, the contractor's design, does not change with changes in the unit cost. The basis solution in Mix 2, on the other hand, is sensitive to the unit price. The basis solution will remain the same (i.e. engineer can use only aggregates and RAP in the mix) so long as 1/2" × 0" aggregate is cheaper than 1/2" × 1/4" and RAP is cheaper than both. This is particularly important since the design will require no purchase of aggregates.

Table 7 Sensitivity analysis of optimal solutions

Parameter	Mix 1			Mix 2		
	Input	Min.	Max	Input	Min.	Max.
1/2" × 0 cost ($/ton)	7.00	-Infinity	Infinity	7.00	2.00	8.50
1/2" × 1/4" cost ($/ton)	8.50	-Infinity	Infinity	8.50	7.00	Infinity
RAP cost ($/ton)	2.00	-Infinity	Infinity	2.00	-Infinity	7.0

The models presented in this work have significant benefits in asphalt mix design. Firstly, the program can quickly and accurately produce aggregate blends that meet specific Bailey method ratios and agency specifications without the trial and error normally involved in such exercises. Secondly, the application of LP optimization techniques assures users that the design is the least cost option that meets all their performance measures. This allows an engineer to manage cost or control stockpile inventories. Finally, this approach, especially if incorporated into a commercial software package (see Awuah-Offei and Askari-Nasab 2009), is likely to encourage the widespread use of the Bailey method in aggregate blending during HMA mix design.

6. Conclusions

HMA mix design is a process of determining the proportions of aggregates and asphalt that will constitute an optimally performing mixture. The role of the aggregate gradation, and consequently the aggregate blend, in HMA mix design is well recognized. The Bailey method provides a structured guideline for developing an aggregate blend which results in HMA mixes that have optimal volumetric properties. This is done through control of three aggregate ratios defined by the method. However, the Bailey method on its own does not remove the trial and error inherent in current mix design techniques. Also, the method does not allow for any means of aggregate cost control or inventory management during HMA mix design. The work presented in this paper applies optimization techniques to the aggregate blending process in HMA mix design and production. HMA aggregate blending has been modeled as a linear programming problem. The models have been verified and validated with real-world examples using Washington State specifications.

The results show that for a 12.5-mm HMA mix designed by a contractor for a WSDOT project, a cheaper aggregate mix could have been obtained using this approach. The optimal mix obtained using this approach is $ 0.74/ton cheaper than the contractor's original mix. The resulting mix is coarser than the original mix with a higher coarse aggregate (CA) ratio (0.103 higher) and a lower ratio of the coarse portion of the fine aggregate (FAc). The increase in CA ratio and decrease in FAc ratio is likely to result in an increase in the voids in mineral aggregate (VMA) and the voids filled with asphalt (VFA) of the HMA. In fact, the VMA of the original HMA was below the recommended 14%. The optimal aggregate mix meets all the WSDOT specifications. The application of these models will allow engineers to quickly develop aggregate mixes for optimal HMA performance without going through the trial and error usually associated with aggregate blending in HMA design. Secondly, this approach provides a means to control HMA aggregate cost and/or aggregate stockpile inventories. It allows engineers to use stockpiles that are abundant and limit the use of those that are limited without sacrificing the performance of the HMA they can produce. Finally, this work will help increase the use of the Bailey method in HMA design and production in the US.

7. Notations

7.1. Symbols

NMPS	NMPS sieve, defined as one sieve larger than the first sieve that retains more than 10% as per SUPERPAVE terminology
PCS	Primary control sieves
SCS	Secondary control sieves
TCS	Tertiary control sieves
CA	Coarse aggregate ratio
FA_c	Coarse portion of the fine aggregate ratio
FA_f	Fine portion of the fine aggregate ratio
c_j	Unit cost ($/ton) of stockpile *j*
x_j	The percentage of stockpile *j* in the mix
n	The number of bins/stockpiles
g_{ij}	The percent passing sieve *i* of stockpile *j*

m	The number of sieve sizes included in the model
l_i	The lower gradation limit for sieve i
u_i	The upper gradation limit for sieve i
a	The sieve number for the half-sieve, b is
b	The sieve number for the primary control sieve
c	The sieve number for the secondary control sieve
d	The sieve number for the tertiary control sieve
i	Sieve number
j	Stockpile number
u	Upper limit
l	Lower limit

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