# Oil sands mine planning and waste management using goal programming

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## Abstract

Strategic mine planning and waste management is an important aspect of surface mining operations. Recent environmental and regulatory requirements makes waste management an integral part of mine planning in the oil sands industry. In oil sands mining, due to the limitation of lease area, the pit phase advancement is carried out simultaneously with the construction of tailings dykes in the mined out areas of the pit. These dykes are constructed to hold tailings that are produced during the processing of the oil sands. Most of the materials used in constructing these dykes come from the oil sands mining operation (overburden and interburden) making it necessary to have a plan for supplying the dyke material. The research problem here is determining the order of extraction of ore, dyke material and waste to be removed from a predefined ultimate pit limit over the mine life that maximizes the net present value of the operation – a strategic schedule. The strategic schedule to be developed is subject to a variety of economic, technical and physical constraints.

We have developed, implemented, and tested a proposed mixed integer goal programming theoretical framework for oil sands open pit production scheduling with multiple material types. The formulation uses binary integer variables to control mining precedence and continuous variables to control mining of ore and dyke material. There are also goal deviational variables and penalty costs and priorities that must be set up by the planner. The optimization model was implemented in TOMLAB/CPLEX environment.

The developed model proved to be able to generate a uniform schedule for ore and dyke material. It also provides the planner the flexibility of choosing goal deviational variables, penalty costs and priorities to achieve a uniform schedule. These parameters can also be used to set priorities for goals thereby leading to improved NPV. Similarly, tradeoffs between achieving a goal and maximizing NPV can be made.

## 1. Introduction

Mining is the process of extracting a beneficial natural resource from the earth (Newman et al., 2010). The extraction process can be an underground or open pit mining operation and this research will be restricted to the latter. An important aspect of mining engineering is mine planning. Whittle (1989) defined open pit mine planning as the process of finding a feasible block extraction schedule that generates the highest net present value (NPV) subject to some operational and technical constraints. Depending on the size of the deposit, the mine plan can be divided into short-term, medium-term and long-term with planning durations ranging from 1 month to 30 years.

Long-term production schedules are the main backbone that drives the activities of the mine throughout its life. The main focus of this research will be on long-term production scheduling

optimization process. The process attempts to maximize the net present value of the overall profit to be generated from the mining operation within some operational and technical constraints such as mining and ore processing capacities, grade blending and block extraction sequencing.

Oil sands mining comprise the mining of overburden material and the McMurray formation. The overburden material is barren and the McMurray formation contains bitumen which is the desirable mineral. About 80% of the oil sands ore after processing finds its way to the tailings dam, making the tailings facility and waste management important aspects of this operation. Due to lack of lease area, these tailings facilities are sited mostly in-pit and embankments or dykes are constructed to contain the tailings. Most of the materials used in constructing these dykes come from the mining operation which makes it necessary to have a plan for supplying the dyke material.

Depending on the dykes' designs, they have different configurations at different locations within the dyke and hence require different material types. Some of the dyke construction methods shown in Fig. 1 are: 1) upstream construction, 2) downstream construction, and 3) centerline construction. More literature that provides details on dyke construction methods for tailings facilities are provided by Vick (1983) and Sego (2010). These dykes are constructed simultaneously as the mine phase advances and the dyke footprints are released. Fig. 2 shows a mining phase advancement schedule at Syncrude Canada Ltd. and this schedule is used to decide the in-pit dyke construction schedule (Syncrude, 2009). This emphasizes the need for a simultaneous development of a life of mine ore and dyke material schedule that can support the mining operation and this will be the main focus of this research.

Currently, scheduling of dyke material is done after mining has started and this may result in inconsistent production of dyke material at different periods during the mine life. It is also a regulatory requirement that life of mine schedules for tailings management strategies are documented and reported annually resulting in the need for a more systematic approach towards oil sands waste management (McFadyen, 2008).

The oil sands mine long-term production planning problem will be modeled numerically as an optimization problem using a mixed integer goal programming model. The optimization problem will be coded using MATLAB (Mathworks Inc., 2009) and solved with TOMLAB/CPLEX which is a large scale mixed integer programming solver. This solver uses a branch and cut algorithm (Holmström, 2009).



Fig. 1. a) Upstream construction, b) downstream construction, and c) centerline construction (after Vick, 1983)



Fig. 2. Syncrude Canada Ltd: North mine development sequence (after Syncrude, 2009)

## 1.1. Oil sands formation

The mineral deposit under consideration is oil sands in the McMurray formation. There are five main soils or rock types associated with this deposit namely:

1) Muskeg/Peat 2) Pleistocene Unit 3) Clearwater Formation 4) McMurray Formation and 5) Devonian carbonates. The oil bearing rock type is the McMurray formation (MMF) which is also made up of three rock types. These are the Upper McMurray (UKM), the Middle McMurray (MKM) and the Lower McMurray (LKM). The main element of interest is bitumen which exists in various grades across the formation. Details of the five rock types associated with the oil sands deposit are:

- 1. Muskeg/Peat this is the topmost overburden material that contains the seeds and roots of native plants and is used for the topmost layer of the reclaimed land. Before mining, this layer is removed and stockpiled and later used for reclamation works.
- 2. Pleistocene Unit (PLU) and Clearwater Formation (CWF) the next rock profiles are the PLU followed by the CWF. These are considered as waste rocks lying above the bitumen bearing McMurray formation. Materials from these profiles are used for road and dyke construction in the mine depending on the soil properties and its mineral content.
- 3. McMurray Formation (UKM, MKM, LKM) the bulk of the bitumen and gas reserves are contained within the McMurray interval in the oil sands area. The McMurray formation rests with profound unconformity on the Devonian carbonates and is unconformably overlain by the Clearwater formation. The McMurray formation ranges between 0 130m thick from Devonian highs to bitumount basins. The LKM is comprised of gravel, coarse sand, silt and clay with siderite as cement. The UKM and MKM comprises of micaceous, fine-to-medium-grained sand, silt and clay, with rare siderite as cement and intraclasts and pyrite nodules up to 10cm in diameter (Hein et al., 2000).
- 4. Devonian Carbonates (DVN) this is the rock type which lies beneath the McMurray formation and is made up of numerous limestone outcrops. It marks the end of the oil sands deposit on a vertical profile.

To obtain details of the oil sands deposit, it is required that a detailed exploration programme is undertaken where drilling is carried out and the resulting data logged for further analysis and modeling. Fig. 3 shows a sketch of the vertical soil profile of an oil sands formation.

Muskeg/Peat
Pleistocene Unit
Clearwater Formation
McMurray Formation
Devonian Carbonates

Fig. 3. Sketch of the vertical soil profile of an oil sands formation

#### 1.2. Oil sands mining and material classification system

The oil sands mining system comprises of the removal of the overburden material and the mining of the McMurray formation. The overburden material comprises of muskeg/peat, the Pleistocene unit and the Clearwater formation. The muskeg/peat, which is barren, is very wet in nature and therefore once it is stripped, it is left for about 2 to 3 years to get it dry, making it easier to handle. This material is stockpiled for future reclamation works which is required for all disturbed landscapes.

The mining of the Pleistocene and Clearwater formation, which is classified as waste, is to enable the exposure of the ore bearing McMurray formation. Some of this material is also used in the construction of dykes and roads required for the operation. The dyke construction is for the development of the tailings dam constructed either in the pit or elsewhere. About 80% of the oil sands mined find its way to the tailings dam after processing. The development of the tailings facility is therefore very important and strategic for the entire oil sands operation.

The classification of the oil sands material is basically driven by economic, technical and regulatory requirements (Dilay, 2001). The geotechnical requirement for dyke construction material varies depending on the dyke design configuration and location of the material within the dyke. The dyke construction material required from the oil sands mining operation (overburden and interburden) must have fines content less than approximately 50%. This type of material contains some amount of clays such as kaolinite, illite, chlorite and smectite (Wik et al., 2007) required as a binding material for improving the stability of the dykes. The waste from the Pleistocene and Clearwater formation is therefore classified based on the fines content. Material with percentage fines less than 50% is classified as dyke material which is required for dyke construction and that with fines more than 50% is classified as waste.

The mining of the oil bearing McMurray formation follows after the mining of the overburden material. By the regulatory and technical requirements, the mineable oil sand grade should be about 7% bitumen content (Dilay, 2001; Masliyah, 2010). All material satisfying this requirement is classified as ore and otherwise as waste. The ore is sent directly to the processing plant for bitumen extraction. This class of waste also known as interburden is reclassified based on the fines content. The material with fines content less than 50% is classified as dyke material and that with fines more than 50% is classified as waste. It is important to note that this material classification system is dynamic and can vary from one mine to another. The criteria may change depending on the grade distribution of the orebody under consideration. Cash flow analysis can also be used to classify the different material types.

The next section of this paper covers a summary of the literature review on mine planning, goal programming, mixed integer programming, oil sands mining, and waste management. Section 3 gives details of the problem definition. The theoretical framework of the proposed formulation is highlighted in section 4. The application of the formulation to an oil sands case with two examples is explained in sections 5 and 6. Conclusions and future research work are outlined in section 7.

## 2. Summary of literature review

The problem of long-term production planning (LTPP) has been a major research area for quite some time now and though tremendous improvements have been made, the current challenging mining environment poses new sophisticated problems. Effective LTPP is critical in the profitability of surface mining ventures and can increase the life of mine considerably. Recent production scheduling algorithms and formulations in literature have been developed along two main research areas: 1) heuristic methods and 2) exact optimization methods (Askari-Nasab and Awuah-Offei, 2009).

Commercial mine scheduling software such as XPAC AutoScheduler (Runge Limited, 2009), WHITTLE (Gemcom Software International, 2008), and NPV Scheduler (Datamine Corporate Limited, 2008) use heuristic methods to generate long-term production schedules. Heuristic methods iterate over different alternatives leading to the generation of the ultimate pit limit with each alternative having a different discounted cash flow and hence NPV of the operation. Due to this, the solution generated may be sub-optimal in terms of NPV.

Authors like Denby and Schofield (1995) and Askari-Nasab (2006) have done extensive research using artificial intelligence techniques to solve the problem of LTPP. Denby and Schofield (1995) used multi-objective optimization to deal with ore grade variance. Using genetic algorithm, Denby and Schofield (1995) tried to maximize value and minimize risk in open pit production planning. Askari-Nasab (2006) also developed and implemented an intelligent-based theoretical framework for open pit production planning. The drawback in the application of these techniques is the non reproducibility of the solution and a measure of the extent of optimality of the solution.

Exact optimization methods have proved to be robust in solving the LTPP problem. They have the capability of considering multiple material types and multiple elements during optimization. This flexibility of mathematical programming models leads to the generation of production schedules with higher NPV than heuristic methods. Goal programming (GP) is an exact optimization technique that has been used for production scheduling in the industry. The advantage of this technique is that it allows for flexible formulation and the specification of priorities among goals or targets. This formulation also allows some form of interaction between the decision maker and the optimization process (Zeleny, 1980; Hannan, 1985). Depending on its use, some alterations are made to the formulation structure. Goal programming was applied to the mine scheduling problem using multiple criteria decision making formulation by Zhang et al. (1993). Multiple goals were considered based on their priorities. The model was tested for a surface coal mine production scheduling and implemented using a branch-and-bound method in 'C' programming language environment. This model was developed for a single ore type process. Chanda and Dagdelen (1995) used goal programming and an interactive graphics system for optimal blending in mine production. Their model sets up the blending problem with multiple goals and attempts to minimize the deviation from the goals using a Fortran 77 computer program based on simplex method of linear programming. The model was tested for a coal mine deposit, but due to some interactions involved in solving the problem, optimal solution cannot be always guaranteed. A mineral dressing criteria was defined by Esfandiri et al. (2004) and used in the optimization of an iron ore mine. A 0-1 non-linear goal programming model was defined based on multiple criteria decision making and the deviations for economics, mining and mineral dressing functions were minimized. This

formulation was solved using LINGO software. The model was found to have limitations and constraints that are numerous for practical application.

Other mine and production related problems have been solved using goal programming with some modifications. Orace and Asi (2004) used a fuzzy goal programming model for optimizing haulage system in an open pit mine. Due to the variations in operating conditions caused by technical, operational, and environmental factors for a mechanical shovel, their model use fuzzy numbers to represent parameters for these operating conditions in optimization. They argue that, their model generates a more realistic results than those based on random numbers derived from probability distributions. A 0-1 goal programming model was developed by Chen (1994) for scheduling multiple maintenance projects for a mineral processing equipment at a copper mine. Using 0-1 decision variables and multiple scheduling periods, the model scheduled for four projects, 40 jobs and nine types of resources. In comparison to a heuristic method that was already used by the mine, the goal programming model reduced the project duration, total project cost and overall workload. Many industrial production planning and project selection decision making problems have been solved making use of the advantages of goal programming formulations (Jääskeläinen, 1969; Mukherjee and Bera, 1995; Leung et al., 2003; Lee et al., 2010).

Other exact optimization techniques that have been used for mine production scheduling are mixed integer programming (MIP) and linear programming (LP). Initial works that was carried out by Johnson (1969), Gershon (1983) and Dagdelen (1985) developed linear programming (LP) and mixed integer programming (MIP) formulations that uses integer variables for optimizing mine schedules. Their formulations could not ensure feasible solutions for all cases and could not overcome the issue of solving large integer programming problems. An integer programming formulation that was developed by Dagdelen and Johnson (1986) uses Lagrangian relaxation and subgradient optimization algorithm to solve the LTPP problem. This formulation could not be implemented on large scale problems and could not handle dynamic cut off grades. Subsequent integer programming models developed by Akaike and Dagdelen (1999) and Caccetta and Hill (2003) use 4D-network relaxation, subgradient optimization algorithm, and branch and cut algorithm respectively to solve the LTPP optimization problem but these models also could not be implemented on large scale problems or handle dynamic cutoff grades.

MIP formulations that was developed by Ramazan and Dimitrakopoulos (2004) attempt to reduce the number of binary variables and solution times by setting certain variables as binary and others as continuous. This resulted in partial mining of blocks that have the same ore value affecting the NPV generated. Ramazan et al. (2005) and Ramazan (2007) developed an MIP model that uses an aggregation method to reduce the number of integer variables in scheduling. This formulation was solved based on fundamental tree algorithm and was used in scheduling a case with 38,457 blocks within the final pit limit. The problem was broken down into four push-backs based on the nested pit approach using WHITTLE (Gemcom Software International, 2008) and formulated as separate MIP models. This would not guarantee a global optimum solution of the problem. Boland et al. (2009) presented an LP approach to generate mine production schedules with block processing selectivity. They however did not present enough information on the generated schedules to enable an assessment of the practicality of the solutions from mining operation point of view.

Recent research work by Askari-Nasab and Awuah-Offei (2009) on the application of exact optimization methods to the LTPP problem has lead to the development of mixed integer linear programming (MILP) models that use block clustering techniques to solve the problem of having large number of decision variables. With a combination of their MILP models and a block clustering algorithm, Askari-Nasab and Awuah-Offei (2009) applied their models to a large scale problem. The formulations use a combination of continuous and binary integer variables. The continuous variables control the portion of a block to be extracted in each period and binary integer variables control the order of block extraction or precedence of mining-cuts through a dependency directed graph using depth-first-search algorithm. The concept of mining-cuts using clustering

techniques is reinforced as an option for solving MILP problems for large scale deposits. The formulation was successfully implemented on an iron ore mine intermediate scheduling case study over twelve periods in TOMLAB/CPLEX (Holmström, 2009) environment. This model does not consider multiple material types.

Due to the advantages that are presented by GP and MILP, some efforts have been made to combine these two techniques and used together for solving industrial problems. This hybrid termed as mixed integer goal programming (MIGP) has been used for scheduling and budgeting problems in nursing, business administration and manufacturing industries (Selen and Hott, 1986; Ferland et al., 2001; Liang and Lawrence, 2007; Nja and Udofia, 2010). MIGP formulation is the proposed model in this paper for application to the oil sands mine LTPP problem.

Oil sands also known as bituminous sands are sedimentary deposits that contain high molar mass viscous petroleum. The largest sources of crude bitumen in the world are Canada and Venezuela (Masliyah, 2010). The origin of the oil that is trapped in the sands to form the oil sands are; marine animals die and sink to the ocean bottom and become embedded by sedimentary minerals. Major alterations caused by aerobic and anaerobic processes, high temperatures and pressures and decomposition produce liquid petroleum. The liquid petroleum flows through the pores of the rock in which it was formed and migrates until it becomes trapped and cannot flow any further, thus forming an oil reservoir (Masliyah, 2010). In the case of the Alberta oil sands, the oil was trapped in the McMurray formation. Oil sands mining started in the 1960s with a surface mining operation that used hot water extraction to recover bitumen from the oil sands and upgrading complex to upgrade the extracted bitumen to a light synthetic crude (Morgan, 2001). This mining operation involves the movement of huge amount of bituminous sands to the processing plant with over 80% being sent to the tailings dam after processing. The remaining waste material mined from the pits are sent to waste dumps or used for dyke construction. This makes waste management an important integral part of the oil sands mining process.

An issue that can bring a mine to its knees within the shortest possible time is the management of its waste. Waste management issues can also result in future unbearable financial liabilities. Strategies for managing oil sands mine waste in an environmentally acceptable manner, in the short and long-term, are a responsibility that cut across a wide range of disciplines. This includes geologists, geotechnical and mine planning engineers, tailings planners, operations and project teams (Fauquier et al., 2009). The team works towards the goal of building tailings dam dykes on time, within budget and design specifications. This involves managing tailings and the general mine waste. Tailings dams that are constructed to store tailings are usually constructed in-pit due to the lack of lease area and the large amount of storage space required. The tailings are stored behind dykes that are constructed in the pit one section at a time as the mining advances. The tailings storage plan requires the in-pit dykes to be designed, constructed and operated on a continual basis throughout the mine life. The in-pit dyke construction materials are derived from the overburden and interburden seams of the deposit (Fauquier et al., 2009). The dyke construction materials are predicted using the geologic block model. This makes it necessary that during the long-term production planning process, schedules for ore and dyke material are generated simultaneously to enable the consistent material supply to the plant and for dyke construction. The nature of dyke material required at any time depends on the dyke configuration and the location of the material within the dyke. A robust oil sands long-term plan should be able to supply ore for the plant and appropriate dyke material throughout the mine life. Fig. 4 shows a typical ore, waste, in-pit dyke and tailings plan for an oil sands mine.

After carefully reviewing the literature on GP, MILP, and MIGP formulations, and oil sands mining and waste management, a formulation has been developed to attempt solving the problem of LTPP for oil sands mining operations. The model has been used to generate a long-term production schedule for ore, dyke material and waste.



Fig. 4. Dyke construction planning at Shell Canada Ltd. – long to medium term (Fauquier et al., 2009)

# 3. Problem definition

Mine management is always faced with the problem of achieving multiple goals with the available limited resources. In oil sands mining, due to the limitation of lease area, the pit phase advancement is carried out simultaneously with the construction of tailings dykes in the mined out areas of the pit, one section at a time. These dykes are constructed to hold tailings that are produced during the processing of the oil sands. Dykes with different configurations are required during the construction. Most of the materials used in constructing these dykes come from the oil sands mining operation (overburden and interburden). It is assumed that the material sent to the processing plant (ore) must have a specified amount of bitumen and percentage fines as well as the material sent for dyke construction (dyke material). Any other material that does not meet the requirements of ore or dyke material is sent to the waste dump.

The main problem here is determining the order of extraction of ore, dyke material and waste to be removed from a predefined ultimate pit limit over the mine life that maximizes the net present value of the operation – a strategic schedule. Fig. 5 shows a schematic representation of the problem definition. Fig. 5 illustrates the scheduling of an oil sands ultimate pit block model containing N blocks. Each block n, is made up of ore  $o_n$ , dyke material  $d_n$ , and waste  $w_n$ . The material in each block is to be scheduled over T periods depending on the goals and constraints associated with the mining operation. For period  $t_1$ , the ore material scheduled is  $o_n^1$ , the dyke material schedule is  $d_n^1$ , and the waste material schedule is  $w_n^1$ .

The strategic schedule to be developed is subject to a variety of economic, technical and physical constraints. The constraints control the mining extraction sequence, ore and dyke material blending requirements and mining, processing and dyke material goals. The mining, processing, and dyke material goals specifies the quantities of material allowed for the mining operation, processing plant and dyke construction respectively.

The strategic schedule is the main driver for the profitability of the oil sands mining operation. The schedule controls the NPV of the operation and enables a robust waste management planning strategy. Improper waste management planning can lead to environmental issues causing immediate mine closure by regulatory agencies and major financial liabilities.



Fig. 5 Schematic representation of the problem definition

## 4. Theoretical framework and models

#### 4.1. Orebody block modeling, ultimate pit limit, and some assumptions

In long-term mine planning, one of the significant steps in the planning process is orebody block modeling. This is made up of the geologic and economic block models which serve as the backbone that drives the activities of the mine throughout its life. It is used for assessing the cash flow of the entire mining operation and then further exploited in subsequent mine planning processes such as optimization, pit design, production scheduling, waste management, equipment selection and plant design. Block models are three-dimensional arrays of rectangular blocks used to model orebodies. Each attribute in a block model represents one characteristic of the volume of material. These attributes include but are not limited to rock types, grades, density and dollar value. These attributes are stored in individual models, which have the same number of rows, columns and levels (Hustrulid and Kuchta, 2006). To determine the size of the block model needed to define the orebody, attributes like the shape and size of the orebody, the mining method, the mining bench height, and mining equipment selection need to be taken into consideration.

The source data for the geologic block model is the information from the drill holes logged from the exploration of the orebody. It is also required that the modeler understands the basic geology of the formation of the orebody and its country or host rock. Cost and revenue information such as mining cost, processing cost and mineral selling price are used in generating the economic block model (Askari-Nasab, 2009).

In geologic modeling, reliable estimation or simulation techniques are provided by Geostatistics to generate block attributes at locations where no data is available. A common estimation technique described as the "workhorse" in Geostatistics is Kriging (Deutsch, 2002). Inverse distance weighting (IDW) is another spatial interpolation algorithm that has been used over the years (ArcGIS, 2010) for geologic modeling. IDW is used for building the geologic model of the

synthetic and real case data used for this initial analysis and Kriging will be used for other real case data in future.

It is assumed that the blocks within the block model are made of smaller regions known as parcels. A parcel is part of a block for which the rock-type, tonnage and element content are known. A block can be made up of zero or more parcels and the total tonnage of the parcels may sum up to the block tonnage or it may be less. The difference, which is waste of unknown rock-type, is known as undefined waste. Neither the location nor the shape of a parcel within a block is defined but the spatial location of each block is defined by the coordinates of its center. Based on the ore tonnage and the grade in each block, the quantity of contained mineral are calculated (Gemcom Software International, 2008; Askari-Nasab and Awuah-Offei, 2009).

It is assumed that the orebody will be extracted using open pit mining techniques and a classical ultimate pit limit design will be generated based on the graph algorithm (Lerchs and Grossmann, 1965; Hustrulid and Kuchta, 2006). This pit outline contains reserves that maximize the profit. As demonstrated by Askari-Nasab and Awuah-Offei (2009), the ultimate pit limit generated directly when an optimal long-term scheduling algorithm is used will become a subset of the conventional ultimate pit limit that is generated using the Lerchs and Grossman's algorithm (Lerchs and Grossmann, 1965). With this basis, the process of finding the optimal long-term strategic and operational schedule will be divided into two steps: 1) determine the ultimate pit limits, and 2) generate a production schedule within the ultimate pit limit.

## 4.2. Clustering

One of the main problems associated with finding the optimal long-term production schedule is that, the size of the problem grows exponentially as the number of blocks increases (Askari-Nasab and Awuah-Offei, 2009) resulting in insufficient computer memory during optimization. This is caused by an increase in the number of decision variables and constraints resulting mainly from the block mining precedence. An efficient way of dealing with this problem is by applying a clustering technique. Clustering is a technique used for aggregating blocks in a block model. In the clustering algorithm to be used in this research, blocks within the same level or mining bench are grouped into clusters based on the attributes: location, rock-type, and grade distribution. These clusters of blocks are referred to as mining-cuts and they have similar attribute definitions to that of the blocks such as coordinates representing the spatial location of the mining-cut.

This block aggregation approach will summarize ore data as well as maintain an important separation of lithology. The total quantity of contained elements in the blocks will be modeled for the mining-cuts to ensure the accuracy of the estimated values. This approach also ensures the planning of a practical equipment movement strategy based on the contained elements and tonnages in the mining-cuts. The clustering algorithm to be used for this research is a fuzzy logic clustering algorithm (Kaufman and Rousseeuw, 1990; Askari-Nasab and Awuah-Offei, 2009).

It is important to note that, clustering of blocks in a block model to mining-cuts reduces the degree of freedom of variables or resolution of the problem when finding the mining sequence that maximizes the NPV of the operation. This may lead to reduced NPV values as compared to a high resolution block level optimization (Askari-Nasab and Awuah-Offei, 2009).

For this paper, since the block model data used has small number of blocks, no clustering was done. Clustering will be done in subsequent implementation of the MIGP model to be conducted on large scale problems.

#### 4.3. The goal programming model

The long-term mine production scheduling problem will be formulated using a combination of mixed integer and goal programming formulation. Using goal programming is appropriate in this context because the structure enables the optimization solution to try achieving a set of goals where some goals can be traded off against one another depending on their priority. Hard constraints can

also be converted to soft constraints which otherwise could lead to infeasible solutions. In simple terms, goal programming allows for flexible formulation and the specification of priorities among goals (Liang and Lawrence, 2007).

The formulated model for the strategic scheduling problem has an objective function, goal function and constraints. The goal objectives are mining, processing and dyke construction. These goals will be prioritized according to the impact of a deviation from their targets on the entire mining operation. The general form of goal programming as applied in multiple criteria decision making optimization can be mathematically expressed as in Eqs. (1) to (4) (Ferland et al., 2001; Esfandiri et al., 2004; Liang and Lawrence, 2007):

**Objective function** 

$$Z = \sum \left( P_i \left( d_i^+ + d_i^- \right) \right)$$

$$P_1 > P_2 > \dots P_i$$
(1)

Goal function

$$\sum C_{ij} X_j + d_i^- + d_i^+ = G_i$$
(2)

Constraints

$$\sum D_{ij}X_j = T_i \tag{3}$$

$$\sum E_{ij} X_j \le M_i$$

Limitations

$$\forall X_i, d_i^+, d_i^- \ge 0 \tag{4}$$

where  $P_i = i$ -th priority

 $X_j$  = decision variable

 $G_i$  = target level of i-th goal

 $T_i$ ,  $M_i$  = mining and technological constraints

 $C_{ij}$  = unit contribution of activity j-th to goal

 $D_{ij}$ ,  $E_{ij}$  = unit contribution of activity j-th to system constraint

 $d_i^+$  = positive deviation

 $d_i^- =$  negative deviation

## 4.3.1 Application of mixed integer goal programming (MIGP) model

## 4.3.1.1. Notation

The notations used in the formulation of the oil sands strategic production scheduling problem has been classified as sets, indices, subscripts, parameters and decision variables.

## 4.3.1.2. Sets

$N = \{1,, N\}$	set of all blocks in the model.
$D_n(J)$	for each block, <i>n</i> , there is a set $D_n(J) \subset N$ which includes all the blocks
	that must be extracted prior to mining block $n$ to ensure that block $n$ is

exposed for mining with safe slopes, where J is the total number of blocks in the set  $D_n(J)$ .

 $C_n(L)$  for each block, *n*, there is a set  $C_n(L) \subset D_n(J)$  defining the immediate predecessor blocks that must be extracted prior to extraction of block *n*, where L is the total number of blocks in the set  $C_n(L)$ .

#### 4.3.1.3. Indices and subscripts

A parameter, f, can take three indices and a subscript in the format  $f_{n,l}^{e,t}$ . Where:

$t \in \left\{1, \dots, T\right\}$	index for scheduling periods.
$n \in \left\{1, \dots, N\right\}$	index for blocks.
$e \in \left\{1, \dots, E\right\}$	index for element of interest in each block.
$l = \{m, p, d\}$	subscripts for mining, processing or dyke construction respectively.
4.3.1.4. Parameters	
$d_n^t$	the discounted profit obtained by extracting block <i>n</i> in period t.
$v_n^t$	the discounted revenue obtained by selling the final product within block $n$ in period $t$ minus the discounted processing cost of all the ore material in block $n$ .
$p_n^t$	the extra discounted cost of mining all the material in block $n$ as dyke material for construction.
$q_n^t$	the discounted cost of mining all the material in block $n$ as waste.
$g_n^e$	the average grade of element $e$ in ore portion of block $n$ .
$\underline{g}^{t,e}$	the lower bound on the required average head grade of element $e$ in period $t$ .
$\frac{-t}{g}$	the upper bound on the required average head grade of element $e$ in period $t$ .
$f_n^e$	the average percent of fines in ore portion of block $n$ .
$\underline{f}^{t,e}$	the lower bound on the required average fines percent of ore in period <i>t</i> .
$\overline{f}^{t,e}$	the upper bound on the required average fines percent of ore in period $t$ .
$f_n^d$	the average percent of fines in dyke material portion of block <i>n</i> .
$\underline{f}^{t,d}$	the lower bound on the required average fines percent of dyke material in period <i>t</i> .
$\overline{f}^{t,d}$	the upper bound on the required average fines percent of dyke material in period <i>t</i> .

<i>O</i> <sub><i>n</i></sub>	the ore tonnage in block <i>n</i> .
<i>W<sub>n</sub></i>	the waste tonnage in block <i>n</i> .
$d_n$	the dyke material tonnage in block <i>n</i> .
$T_m^t$	the mining goal in period <i>t</i> (tonnes).
$d_1^{-,t}$	the negative deviation from the mining goal in period $t$ (tonnes).
$d_1^{+,t}$	the positive deviation from the mining goal in period $t$ (tonnes).
$T_p^t$	the processing goal in period t (tonnes).
$d_2^{-,t}$	the negative deviation from the processing goal in period $t$ (tonnes).
$d_2^{+,t}$	the positive deviation from the processing goal in period $t$ (tonnes).
$T_d^t$	the dyke material goal in period t (tonnes).
$d_{3}^{-,t}$	the negative deviation from the dyke material goal in period $t$ (tonnes).
$d_3^{+,t}$	the positive deviation from the dyke material goal in period $t$ (tonnes).
$P_1$	the priority level associated with minimizing the deviations from the mining goal.
$P_2$	the priority level associated with minimizing the deviations from the processing goal.
$P_3$	the priority level associated with minimizing the deviations from the dyke material goal.
$r^{e,t}$	the proportion of element $e$ recovered in time period t (processing recovery).
$p^{e,t}$	the price of element <i>e</i> in present value terms per unit of product.
S <sup>e,t</sup>	the selling cost of element $e$ in present value terms per unit of product.
$c^{e,t}$	the cost in present value terms per tonne of ore for processing.
$k^{t}$	the cost in present value terms per tonne of dyke material for dyke construction.
$m^{t}$	the cost in present value terms of mining a tonne of waste in period t.
$a_1$	the penalty paid per tonne in deviating from the mining goal.
<i>a</i> <sub>2</sub>	the penalty paid per tonne in deviating from the processing goal.
<i>a</i> <sub>3</sub>	the penalty paid per tonne in deviating from the dyke material goal.
4.3.1.5. Decision	variables
$x_n^t \in [0,1]$	a continuous variable representing the portion of block <i>n</i> to be extracted as ore and processed in period <i>t</i> .

 $z_n^t \in [0,1]$  a continuous variable representing the portion of block *n* to be extracted as dyke material and used for dyke construction in period *t*.

- $y_n^t \in [0,1]$  a continuous variable representing the portion of block *n* to be mined in period *t*, which includes both ore, dyke material and waste.
- $b_n^t \in [0,1]$  a binary integer variable controlling the precedence of extraction of blocks.  $b_n^t$  is equal to one if the extraction of block *n* has started in period t, otherwise it is zero.

#### 4.3.1.6. Modeling of economic block value

The objective function of the MIGP model for strategic LTPP is to maximize the net present value of the mining operation and minimize the deviations from the mining goal, processing goal and dyke material goal. The concept of economic block value is based on ore parcels which could be mined selectively. The profit from mining a block is a function of the value of the block and the cost incurred in mining, processing and dyke construction. The discounted profit from block n is equal to the discounted revenue obtained by selling the final product contained in block n minus the discounted cost involved in mining block n minus the extra discounted cost of mining dyke material (Askari-Nasab and Awuah-Offei, 2009). This has been simplified into Eqs. (5) to (8).

$$d_n^t = v_n^t - q_n^t - p_n^t \tag{5}$$

Where:

$$v_n^t = \sum_{e=1}^E o_n \times g_n^e \times r^{e,t} \times \left( p^{e,t} - s^{e,t} \right) - \sum_{e=1}^E o_n \times c^{e,t}$$
(6)

$$q_n^t = \left(o_n + d_n + w_n\right) \times m^t \tag{7}$$

$$p_n^t = d_n \times k^t \tag{8}$$

#### 4.3.1.7. The mixed integer goal programming model

Using multiple criteria decision making analysis, the objective functions of the MIGP model for strategic LTPP as applied in oil sands mining, can be formulated as maximizing the NPV and minimizing deviations from the goals. These are represented by Eqs. (9) and (10).

$$Max \sum_{t=1}^{T} \sum_{n=1}^{N} \left( v_n^t \times x_n^t - p_n^t \times z_n^t - q_n^t \times y_n^t \right)$$

$$\tag{9}$$

$$Min\sum_{t=1}^{T}\sum_{n=1}^{N} \left[ P_1\left(a_1d_1^{-,t}\right), P_2\left(a_2d_2^{-,t}\right), P_3\left(a_3d_3^{-,t}+a_3d_3^{+,t}\right) \right]$$
(10)

Eqs. (9) and (10) can be combined as a single objective function formulated as in Eq. (11).

$$Max \sum_{t=1}^{T} \sum_{n=1}^{N} \left[ \left( v_n^t \times x_n^t - p_n^t \times z_n^t - q_n^t \times y_n^t \right) - \left( P_1 \left( a_1 d_1^{-,t} \right) + P_2 \left( a_2 d_2^{-,t} \right) + P_3 \left( a_3 d_3^{-,t} + a_3 d_3^{+,t} \right) \right) \right]$$
(11)

The complete MIGP model comprising of the objective function, goal function and constraints can be formulated as:

Objective function:

$$Max \sum_{t=1}^{T} \sum_{n=1}^{N} \left[ \left( v_n^t \times x_n^t - p_n^t \times z_n^t - q_n^t \times y_n^t \right) - \left( P_1 \left( a_1 d_1^{-,t} \right) + P_2 \left( a_2 d_2^{-,t} \right) + P_3 \left( a_3 d_3^{-,t} + a_3 d_3^{+,t} \right) \right) \right]$$
(12)

Goal functions:

$$\sum_{n=1}^{N} \left( \left( o_n + w_n + d_n \right) \times y_n^t \right) + d_1^{-,t} = T_m^t \qquad \forall t \in \{1, ..., T\}$$
(13)

$$\sum_{n=1}^{N} \left( o_n \times x_n^t \right) + d_2^{-,t} = T_p^t \qquad \forall t \in \{1, ..., T\}$$
(14)

$$\sum_{n=1}^{N} \left( d_n \times z_n^t \right) + d_3^{-,t} - d_3^{+,t} = T_d^t \qquad \forall t \in \{1, ..., T\}$$
(15)

Constraints:

 $x_n^t + z_n^t \le y_n^t$ 

$$\underline{g}^{t,e} \leq \sum_{n=1}^{N} g_n^e \times o_n \times x_n^t / \sum_{n=1}^{N} o_n \times x_n^t \leq \overline{g}^{t,e} \qquad \forall t \in \{1,...,T\}, \quad e \in \{1,...,E\}$$

$$(16)$$

$$\underline{f}^{t,e} \leq \sum_{n=1}^{N} f_n^e \times o_n \times x_n^t \bigg/ \sum_{n=1}^{N} o_n \times x_n^t \leq \overline{f}^{t,e} \qquad \forall t \in \{1,...,T\}, \quad e \in \{1,...,E\}$$

$$(17)$$

$$\underline{f}^{t,d} \leq \sum_{n=1}^{N} f_n^d \times d_n \times z_n^t \Big/ \sum_{n=1}^{N} d_n \times z_n^t \leq \overline{f}^{t,d} \qquad \forall t \in \{1,...,T\}, \quad d \in \{1,...,D\}$$
(18)

$$\forall t \in \{1, ..., T\}, n \in \{1, ..., N\}$$
 (19)

$$b_n^t - \sum_{i=1}^t y_s^i \le 0 \qquad \forall t \in \{1, ..., T\}, \ n \in \{1, ..., N\}, \ s \in C(L) \qquad (20)$$

$$\sum_{i=1}^{t} y_{n}^{i} - b_{n}^{t} \le 0 \qquad \forall t \in \{1, ..., T\}, \quad n \in \{1, ..., N\}$$
(21)

$$b_n^t - b_n^{t+1} \le 0$$
  $\forall t \in \{1, ..., T-1\}, n \in \{1, ..., N\}$  (22)

$$d_1^{-,t}, d_2^{-,t}, d_3^{-,t}, d_3^{+,t} \ge 0 \qquad \forall t \in \{1, \dots, T\}$$
(23)

$$P_1 > P_2 > P_3 \tag{24}$$

Eq. (12) is the objective function of the formulation which seeks to maximize the net present value and minimize the deviations from the mining, processing and dyke construction material goals. Eqs. (13), (14), and (15) are the goal functions which define the mining, processing and dyke material goals that are required to be achieved. Eqs. (16), (17), and (18) specify the limiting requirements for ore bitumen grade, ore fines and dyke material fines. Eq. (19) ensures that the total material mined in each period does not exceed the sum of the ore and dyke material mined. Eqs. (20), (21) and (22) check the set of immediate predecessor blocks that must be mined prior to mining block n. Eq. (23) ensures that the negative and positive deviations from the targeted mining, processing and dyke material goals are always positive. Eq. (24) states the order of prioritization associated with achieving the goals. The model assumes that there exists a pre-emptive priority structure among the goals and this can be changed depending on the aim of optimization.

#### 5. Application of the MIGP model for strategic production scheduling

The problem being looked at here is finding the sequence of extracting ore, dyke material and waste blocks from a predefined ultimate pit over the mine life so that the NPV of the operation is

maximized. The production schedule is subject to some physical, technical and economic constraints such as mining extraction sequence and mining and processing capacities.

The formulation presented here for open pit strategic production scheduling has the objective of maximizing the NPV of the mining operation and minimizing the goal deviations whilst achieving the multiple goals of ore and dyke material requirements. The block sizes used for production scheduling must be similar to the selective mining units; otherwise, the generated schedule may not be feasible in practice. The formulation has been implemented and tested with different sets of parameters for calibration. The proposed MIGP model uses binary integer decision variables,  $b_n^t$  to control precedence of block or mining-cut extraction and continuous variables,  $y_n^t$ ,  $x_n^t$ , and  $z_n^t$  are used to model extraction, processing and dyke material requirements respectively at block or mining-cut level. Continuous deviational variables,  $d_1^{-,t}$ ,  $d_2^{-,t}$ ,  $d_3^{-,t}$ , and  $d_3^{+,t}$  have been defined to support the goal functions that control extraction, processing and dyke material. These deviational variables provide a continuous range of units (tonnes) that the optimizer chooses from to satisfy the defined goals. The deviational variables are minimized in the objective function. Within the objective function, deviational penalty cost parameters have been defined by  $a_1$ ,  $a_2$ , and  $a_3$  which penalizes the NPV for any deviation from the set goals. This is a significant component of this formulation in that, the optimizer is forced to meet the set goals in order to avoid being penalized resulting in an improved NPV value. To be able to place more emphasis on the goals which are more important, priority parameters  $P_1$ ,  $P_2$ , and  $P_3$  have been defined in the objective function which also penalize the NPV more if the most important set goal is not met. This ensures that goals such as processing are prioritized above dyke material requirements to improve the NPV.

## 5.1. Precedence of block/mining-cut extraction

The mining precedence constraints are the main reason for the increase in the number of constraints and the complexity of the scheduling problem. Previous mining precedence constraint set-ups enforces that, before a block is mined all blocks on top of it must be mined first. For the example shown in Fig. 6, before block 1 is mined, the 24 blocks above it must be mined first. This results in 24 mining precedence constraint equations in each period for block 1. This increases the size of the problem quickly making the optimization intractable (Askari-Nasab and Awuah-Offei, 2009).

The mining precedence constraint in the MIGP formulation has been modeled using the directed graph theory (Askari-Nasab and Awuah-Offei, 2009). Eqs. (20) to (22) in the model control the relationship of block extraction precedence by the binary integer variable,  $b_n^t$ . Specifically, Eq. (20) ensures that only the set of immediate predecessor blocks on the top of a block need to be extracted prior to extracting the block. This is represented by the set  $C_n(L)$  in the formulation. Eq. (21) ensures that if extraction of block *n* is started in period *t*, then block *n* has not been extracted before. Eq. (22) ensures that  $b_n^t$  is equal to one if extraction of block *n* has started by or in period *t*, otherwise it is zero. This means once the extraction of a block starts in period *t*, this block is available for extraction during the subsequent periods. Fig. 6 shows the set that has to be mined for extracting block 1:  $C_1(L) = \{2, 3, 4\}$ . This results in 3 mining precedence constraint equations in each period for block 1 as compared to 24 in previous formulations. This decreases the size of the mine production scheduling optimization problem considerably.



Fig. 6. Block extraction precedence in the MIGP formulation

#### 5.2. Selecting deviational, penalty and priority parameters in the model

To select the appropriate goal deviational parameters for a given problem, the modeler is required to first select a large continuous variable range starting from zero. This is to ensure that the optimizer can find an initial solution for the problem at hand and to give the modeler an idea of which periods certain types of material are available subject to the physical, technical and economic constraints imposed on the schedule. With this background knowledge, the modeler can start tightening the range of the deviational variables for specific goals in some periods. This is to ensure that the solution of the optimization problem is found with as little deviation as possible from the set goals.

The initial penalty cost for deviating from the set goals are set to zero. This literally means that, no penalties are being applied initially for deviating from the set goals. The NPV of the operation generated at this point serves as the baseline NPV which can be improved upon. The next step is to set penalty cost for deviating from the ore and dyke material goals. No penalty cost should be set for deviating from the mining goal because after mining ore and dyke material, the optimizer will be forced to mine waste to make up for the mining goal thereby reducing the NPV of the mining operation. The deviational penalty cost for ore and dyke material should be set below the average cash flow for mining a unit (tonne) of ore or dyke material. This ensures that the optimizer avoids going in for a deviational variable to make up the set goals unless it has no other option under the prevailing constraints. This stems from the fact that in the objective function, the deviation from the goals are being minimized and any deviation from the goals will have a negative impact on the NPV as a result of the penalty cost associated with the deviations. This makes deviational variables chosen by the optimizer in each period as low as can guarantee a feasible solution. If the deviational penalty cost for ore or dyke material are set above the average cash flow for mining ore or dyke material, the optimizer may in some periods choose to go in for deviational variables to make up the set goals since it sees that as a less expensive option to have a greater NPV than mining the required material to get the set goals. The subsequent deviational penalty cost for ore and dyke material should be increased further and the schedule re-optimized to see its impact on the NPV. An increase in the NPV resulting from mining more ore is expected. Subsequent increase in the deviational penalty cost may also result in a decrease in NPV if the optimizer has to mine more waste before mining the ore required to make up the goal; resulting in a negative cash flow for that additional ore block.

The element of goal priority setting in the formulation was introduced to enforce the priority of one goal over the other. This is introduced in the formulation in a similar way to that of penalty cost. Deviational variables of goals with higher priorities are made to carry a heavier negative impact on the NPV of the operation than goals with lower priorities. This forces the optimizer to ensure that goals with higher priorities deviate least from the set goals to avoid higher negative impacts on the

NPV of the operation. For example, it is obvious that in oil sands mining, the priority for achieving goals for processing will be set higher than dyke material.

Whilst setting up these parameters, the modeler needs to keep an eye on the NPV to become aware of the impact of any parameter change on the NPV. It is interesting to note that in some cases the extent of setting the penalty cost below the average cash flow of mining ore or dyke material depends on the extent to which the modeler is ready to trade off NPV to meet the set goals. For example, placing a much higher penalty on ore goal deviational variable may result in a negative cash flow, depending on how many waste or dyke material blocks need to be mined before getting access to an ore block to make up the set goal. In such a case, a higher penalty may enforce the ore goal to be met whilst reducing the NPV of the operation.

In summary, the relationship between penalty cost and priority variable for ore and NPV of the operation is not linear. A general value function plot of NPV of the operation versus penalty cost and priority variable for ore shows an initial direct relationship which later becomes an inverse relationship. This is shown in Fig. 7. The details for the value function plot vary according to the problem setup. It can also be seen from Fig. 7 that the demand for ore in relation to the NPV is not open ended. This is in line with the statement made by Hannan (1985) that in goal programming "more is not always better despite the fact that it is represented that way in the objective function".





#### 5.3. Solving the optimization problem

The use of the MIGP formulation for an orebody model usually results in a large scale optimization problem. One of the recent optimization solvers capable in handling such problems is the ILOG CPLEX (ILOG Inc., 2007). This optimizer was developed based on branch and cut algorithm and makes the solving of MIGP models possible for large scale problems. Branch and cut is an algorithm which combines different optimization techniques for solving integer programming problems. This algorithm is a hybrid of branch-and-bound and cutting plane methods (Horst and Hoang, 1996; Wolsey, 1998).

This research uses TOMLAB/CPLEX (Holmström, 2009) as the MIGP model solver. The CPLEX solver engine (ILOG Inc., 2007) has been successfully integrated with MATLAB (Mathworks Inc., 2009) working environment by TOMLAB/CPLEX. An optimization termination criterion that the user set in CPLEX is the gap tolerance (EPGAP). The gap tolerance sets an absolute tolerance on the gap between the best integer objective and the objective of the best node remaining. It instructs CPLEX to stop as soon as it has found a feasible integer solution proved to be within the set EPGAP.

## 6. Results and discussions

We have developed, implemented, and tested the proposed MIGP model presented in section 4.3 in TOMLAB/CPLEX environment (Holmström, 2009). The performance of the proposed model is analyzed based on net present value, mining production goals and smoothness of the generated schedules. The model is verified by numerical experiments on a synthetic data set containing 120 blocks on four mining benches and a real mining ultimate pit containing 555 blocks on five mining benches. This was for an oil sand deposit and scheduling was done for ore, dyke material and waste over 15 periods. The model was tested on a Dell Precision T3500 computer at 2.4GHz, with 3GB of RAM. A relative tolerance of 1% and 5% on the gap between the best integer objective and the feasible integer solution was set for the 120 blocks and 555 blocks respectively.

Table 1 and Table 2 show the inputs and numerical results of the test of the MIGP model with the data set containing 120 blocks over five periods of extraction. The initial goal deviational variables were set at 20,000 tonnes. This was subsequently modified and the final goal deviational variables were set at 20,000 tonnes for mining and processing goals and 300 tonnes for dyke material goal for all the runs. In setting the deviational variables, among other things the smoothness of the schedule should be monitored. Initially, the model was run with no penalty cost and goal priorities to get the baseline NPV which is subject to further improvement. Subsequent penalty cost and priorities placed on the goals did not bring any further NPV improvements. This was because all the 120 blocks were mined either with or without penalty cost and priorities. The average runtime was 4 seconds. The required ore and dyke material goals were met within the set deviational variables as well as the grade of each material. Figs. 8, 10, and 11 show the schedules for ore, dyke material and waste over the five periods for run 3. Fig. 9 shows the average ore grade over the five periods.

Run	Penalt	y cost (\$/tonne	) and priority	Goals (Million tonnes)			
Itun	Mining Processing Dy		Dyke material	Mining	Processing	Dyke material	
1	0	0	0	1.95	1.35	0.52	
2	0	3	2	1.95	1.35	0.52	
3	0	6	4	1.95	1.35	0.52	

Table 1. Inputs for the synthetic data containing 120 blocks scheduled over 5 periods

Table 2. Results for the synthetic data containing 120 blocks scheduled over 5 periods

Run	Root Node Gap %	NPV (\$M)
1	0.45	1.21
2	5.53	1.21
3	7.91	1.21



Fig. 8. Tonnage of ore, dyke material and waste per period



Fig. 9. Average ore grade (bitumen %) per period

8	8	8	8	8	8	8	8	
8	4	4	3	5	5	5	8	
8	3	3	3	4	4	4	8	<sup>50m</sup>
8	2	3	3	4	4	4	8	\$0m
8	2	3	3	4	2	2	8	
8	2	2	1	4	4	4	8	
8	3	3	3	5	5	5	8	
8	8	8	8	8	8	8	8	

Plan View - 1530m

Fig. 10. Plan view of bench 1575m showing periods of block extraction

#### Cross section looking east - 98400m



Fig. 11. Cross section 98400m looking east showing periods of block extraction

Table 3 shows inputs and numerical results from the MIGP model for an oil sands ultimate pit data set containing 555 blocks over 15 scheduling periods. The ultimate pit contains five 15m benches from elevation 312.5m to 237.5m. The ultimate pit was obtained using WHITTLE (Gemcom Software International, 2008). Each block represents a volume of 100m x 100m x 15m of rock. The original block model was reblocked by putting four blocks into one, thereby resulting in 555 blocks with four parcels each in the ultimate pit. The model contains 178 million tonnes of material with 94 million tonnes of ore. The ore bitumen grade ranges from 7 to 15.5% with an average grade of 10.8%. The ore fines grade ranges from 0 to 30.0% with an average grade of 6.8%. It also contains 70 million tonnes of dyke material. The dyke material fines grade ranges from 0 to 47.2% with an average grade of 1.1%. Bitumen and fines grades need to be controlled within an acceptable range for processing plant feed and dyke construction.

Run	Penalt	y cost (\$/tonne	and priority	Root Node	NPV (\$M)	
Run	Mining	Processing	Dyke material	Gap %		
1	0	0	0	2.60	336.93	
2	0	4	2	2.32	337.91	
3	0	10	6	2.67	\$339.49	
4	0	16	8	4.78	331.80	

Table 3. Inputs and results for the oil sands data set containing 555 blocks scheduled over 15 periods

Run	Mining goals (Million tonnes)			Processing goals (Million tonnes)			Dyke material goals (Million tonnes)		
	P1-P3	P4-P9	P10-P15	P1-P3	P4-P9	P10-P15	P1-P3	P4-P9	P10-P15
1, 2, 3, 4	10.5	10.5	13.9	0	6.5	8.5	10.5	3.8	2.6

Table 4. Inputs for the oil sands data set containing 555 blocks scheduled over 15 periods

Table 5. Inputs for the oil sands data set containing 555 blocks scheduled over 15 periods

Run	Mining deviation (Million tonnes)			Processing deviation (Million tonnes)			Dyke material deviation (Million tonnes)		
	P1-P3	P4-P9	P10-P15	P1-P3	P4-P9	P10-P15	P1-P3	P4-P9	P10-P15
1, 2, 3, 4	1.0	1.0	1.0	0	0.05	0.5	0.1	0.1	0.1

Table 6. Inputs for the oil sands data set containing 555 blocks scheduled over 15 periods

Run	Ore bitum (wt	en grade %)	Ore fine (wt	es grade %)	Dyke material fines grade (wt%)		
	$\frac{-t,e}{g}$	$\underline{g}^{t,e}$	$\overline{f}^{t,e}$	$\underline{f}^{t,e}$	$\overline{f}^{t,d}$	$\underline{f}^{t,d}$	
1, 2, 3, 4	12	7	23	0	50	0	

It is desired to keep an average processing plant head grade with bitumen content between 7 and 12% and fines content less than 23%. The dyke material is required to have bitumen content less than 7% and fines content less than 50%. Our goal is to generate a uniform schedule based on the availability of material, the plant processing and dyke construction requirements. Further to this, we intend to keep a steady stripping ratio of about 1.6 when the mining of ore starts. This would ensure that the mining equipment capacity will be uniform over some time. Table 6 shows the input grade limits set for ore and dyke material.

To schedule the 555 blocks over 15 periods, the initial deviational variables chosen were 1M tonnes for all the goals. These were further reduced based on the availability of material to enable a uniform and smooth supply of plant feed and dyke construction material and maintain a uniform mining capacity. The mining, processing and dyke material goals were also modified accordingly. The final goal deviational variables used for the runs are shown in Table 5. The associated goals are shown in Table 4.

The initial penalty cost and priority were set to zero to establish the baseline NPV. These were subsequently increased and the NPV recalculated whilst monitoring the uniformity and smoothness of the resulting schedule. The NPV starts to increase due to the deviational penalty cost and priority placed on the ore material. The NPV reaches some threshold value under the set goals and constraints and any further increase in the deviational penalty cost and priority results in a decrease in the NPV. This happens when the penalty cost and priority placed on the ore material is so high that, whilst forcing the optimizer to mine more ore to achieve the goal, the cash flow from any

additional ore block is negative. This may result from an ore block being overlain by lots of waste or dyke material which must be mined before mining the ore block. This drives the overall value of mining the additional ore block negative. From Table 3, run 3 generated the maximum NPV and a uniform schedule and the results that follow are based on this run.

An important feature of this MIGP formulation, that makes it a robust and flexible platform for mine planning, is that apart from the NPV maximization, the planner has control over the setting of goals and their deviational variables, upper and lower limits of grades and the deviational penalty cost and priority of goals. The planner can also decide on tradeoffs between NPV maximization and goal achievement.

The results from run 3 shows that during the first three years, due to the formation of the oil sands deposit, dyke material was the most abundant as a result of the presence of the overburden material which must be stripped. Once the overburden material is mined, ore becomes available in period 4 and was again ramped up in period 10. The dyke material was ramped down after period 3 and reduced further in period 10 due to the dwindling reserves of interburden dyke material. The mining capacity was uniform for the first 9 periods and then increased for the remaining periods. Figs. 12, 13, 14, and 15 show the schedules for ore, dyke material, and waste mined over the 15 scheduling periods. The total amount of material mined was 176 million tonnes. This is made up of 87 million tonnes of ore, 68 million tonnes of dyke material and the rest being waste. It can be seen from the schedule that the mining of waste was delayed until later years. This phenomenon is in accordance with the objective of maximizing the NPV of the operation.





Fig. 12. Tonnage of ore, dyke material and waste per period



Fig. 13. Plan view of bench 297.5m showing periods of block extraction

#### Cross section looking east - 47800m



Fig. 14. Cross section 47800m looking east showing periods of block extraction

## Cross section looking north – 50800m



Fig. 15. Cross section 50800m looking north showing periods of block extraction

It is our target to blend the run-of-mine ore to meet the quality and quantity specification of the processing plant and dyke construction. As more detailed planning is done in the short and medium term, the blending problem becomes more prominent. The plant head grade and the dyke material grade that was set were successfully achieved in all the periods. The ore material bitumen grade was between 9.5 and 12% with fines grade between 5% and 17%. The dyke material fines grade was also between 0 and 50%. These can be seen in Figs. 16, 17, and 18. It can be seen in the graphs that there is a general trend of decreasing bitumen grade for ore over the periods. This high grading phenomenon in the early periods is in accordance with the objective of maximizing the NPV of the operation.



Fig. 16. Average bitumen grade of ore per period



Fig. 17. Average fines grade of ore per period



Fig. 18. Average fines grade of dyke material per period

#### 7. Conclusions and future research work

This paper discussed the current application of mixed integer programming and goal programming to the open pit long-term production scheduling problem and some of the limitations. Further to this, we looked at the use of mixed integer goal programming formulations in solving industrial problems due to the advantages derived from this hybrid formulation. In this paper, we have developed, implemented and tested a MIGP optimization formulation for open pit production scheduling of multiple material types. The oil sands mining case was used. This requires that a production schedule is generated for ore, dyke material and waste. These schedules ensure that whilst ore is fed to the plant, there is enough material available for dyke construction. This enables adequate space for the in-pit storage of tailings from the plant.

The formulation uses binary integer variables to control mining precedence and continuous variables to control mining of ore and dyke material. There are also goal deviational variables and penalty costs and priorities that must be set up by the planner. The optimization model was implemented in TOMLAB/CPLEX environment.

The developed model proved to be able to generate a uniform schedule for ore and dyke material. It also provides the planner the flexibility of choosing goal deviational variables, penalty costs and priorities to achieve a uniform schedule. These parameters can also be used to set priorities for goals thereby leading to improved NPV. Similarly, tradeoffs between achieving a goal and maximizing NPV can be made.

From the results of the schedule, there is a problem where the block extraction sequence, especially for the first bench, does not follow a practical mining sequence. This is due to the degree of freedom for the first bench blocks which are not constrained in any way during optimization in terms of mining precedence thereby leading to a schedule that is less smooth. Further research will focus on developing a horizontal direction block extraction sequence constraint that will enforce a more systematic way of mining especially for the first bench. This becomes more significant in the short term due to the equipment maneuverability requirements in oil sands mining. Clustering will also be done for large scale deposits.

## 8. Appendix

Matlab code developed to define and implement the proposed MIGP formulation for oil sands production scheduling optimization.

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