Incorporation of Parametric Uncertainty into the Gasoline Blending Control problem

“In these matters the only certainty is that nothing is certain.” - Pliny the Elder

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Outline

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  - Gasoline Blending
  - Modeling of the blending process
- Uncertainty in blending
- Conventional blending controllers*
- Proposed formulation for Gasoline Blending Control
- Results
- Conclusions & future work

* and the not so conventional
Gasoline Blending Process

- Feedstocks (with desirable properties) from upstream processes are blended.
- Strict specifications on the qualities of the blended gasoline.
- Octane number (RON, MON), and Reid Vapour Pressure (RVP) are some typical quality specifications.
- Blended from standing or running tanks.
Modeling and Control of the blending process

Blending Models

\[ q_{i,\text{blend}} \times \sum_{j=1}^{n} x_j = g_i \left( q_{i,j} x_j \right) \]

- \( g_i \) can be a linear or non-linear function for the \( i^{th} \) quality of the blend (\( q_{i,\text{blend}} \)).
- \( j \) is the feedstock index for the \( i^{th} \) quality/property.
- \( n \) is the number of feedstocks.

The control algorithm

\[
\begin{align*}
\min_x & \quad c^T x \\
\text{subject to :} & \quad g(Q, x) \leq s(1^T x) \\
& \quad Hx \leq b
\end{align*}
\]

- \( c \): cost vector combining the cost of the feedstocks and the price of the blended gasoline
- \( x \): vector of feedstock flows
- \( Q \): matrix of feedstock qualities
- \( s \): vector of quality specifications
- \( H \): matrix for (linear) demand and availability constraints
- \( b \): vector of availabilities and demand
Uncertainty in constraint parameters

- Uncertainty in any/all of the qualities of the feedstocks can lead to calculation of infeasible (violation of specifications) setpoints for the feedstock flows.
- Infeasible blends have to be reblended to bring them back up to specifications.
- Causes of uncertainty:
  - upstream process (FCCU, Cat Reformer, Atmospheric Column, etc.) changes
  - measurement errors from quality sensors
Conventional Blending Controllers

RTO based controllers with “bias” updating (Singh, 1996)

\[
\begin{align*}
\min_{x} & \quad c^T x \\
\text{subject to :} & \quad g(Q,x) \leq s(1^T x) - \beta \\
& \quad Hx \leq b
\end{align*}
\]

\( \beta \) is a vector of bias terms which is updated at each optimization interval.
- It is calculated as the difference between measured blended qualities and those predicted by the model.
- Designed for step type disturbances (Forbes & Marlin, 1994).
Gasoline Blending Controllers contd.

Time-Horizon Controller (Singh, 1996)

$$\min_{x} \int_{t_p}^{t_f} (c^T x) dt$$

subject to:

$$\int_{t_o}^{t_p} g(Q, x) dt + \int_{t_p}^{t_f} g(\hat{Q}, x) dt \leq \left\{ \int_{t_o}^{t_f} (1^T x) dt \right\} s$$

$$g(\hat{Q}_{t_p}, x_{t_p}) \leq (1^T x_{t_p}) s$$

$$\int_{t_o}^{t_f} (Hx) dt \leq b$$

- Feedstock qualities are predicted over a user-specified horizon and control moves are calculated from the present to the end of the blend.
- The second constraint tries to ensure that each blend section is feasible by itself.

Advantages:

- Compensates for past measured off-specification blend sections.
- Pre-compensates for anticipated trends in feedstock qualities.
**Introduction to Stochastic Programming**

“Stochastic Programming handles mathematical problems where some of the parameters are random variables …” - Andras Prekopa

**General optimization problem**

\[
\min_x f(c,x) \\
\text{subject to:} \\
g(\xi, x) \geq b
\]

- Based on the above, Stochastic Programming can be split into three categories:
  - uncertainty in the objective function \(c\)
  - uncertain right hand sides of constraints \(b\)
  - uncertain parameters in \(g(\xi)\)

\[\text{In this work only the third category is considered: uncertain } \xi\]
Stochastic Programming for uncertain constraints

- **Probability maximization**
  \[
  \max_x \quad \text{Prob}\{ g(x, \xi) \geq b \}
  \]

- **Probabilistic constraints or Chance Constrained Programming**
  \[
  \min_x \quad f(c, x) \quad \text{and} \quad \min_x \quad f(c, x) \\
  \text{subject to:} \quad \text{subject to:} \\
  \text{Prob} \{ g_i(\xi, x) \geq b_i \} \geq \pi_i \\
  \text{Prob} \{ g(\xi, x) \geq b \} \geq \pi
  \]

- **Constraints with conditional expectations**
  \[
  \min_x \quad f(c, x) \\
  \text{subject to:} \\
  E\{ b - g(\xi, x) \mid g(\xi, x) < b \} \leq d \\
  d \Rightarrow \text{vector of maximum violation}
  \]

- **Penalty function form**
  \[
  \min_x \quad f(c, x) + E\{ p(b - g(\xi, x)) \} \\
  p \Rightarrow \text{penalty function on constraint violation}
  \]
**CCP in Gasoline Blending Control**

**Dynamic Time-Horizon form**

\[
\min_x \sum_{i=p}^{n} c^T x_i
\]

subject to:

\[
\text{Prob}\left\{ \sum_{i=1}^{p-1} \overline{Q_i} x_i + \sum_{i=p}^{n} \widehat{Q_i} x_i \leq s_i \sum_{i=1}^{n} (1^T x_i + l_i) \right\} \geq \pi \Rightarrow \pi \text{ is a vector}
\]

\[
\sum_{i=p}^{n} H x_i \leq b
\]

\[
\min_x \sum_{i=p}^{n} c^T x_i
\]

subject to:

\[
\text{Prob}\left\{ \left(\overline{Q}_{p_2n} - s1_{p_2n}^T - l_{p_2n}\right) x_{p_2n} \leq -\sum_{i=1}^{p-1} (\overline{Q_i} - s1^T - l_i) x_i \right\} \geq \pi
\]

\[
H_{p_2n} x_{p_2n} \leq b
\]

Similar to blend-horizon controller except for inclusion of chance constraints for uncertainty around predicted qualities.
Case study description

• All qualities of all feedstocks are uncertain.
• The noise model is given by:

\[
y_k = \left( \frac{1}{1 - z^{-1}} \right) \left( 1 - 0.56 z^{-1} \right) w_k = \frac{0.44 w_{k-1}}{1 - 1.56 z^{-1} + 0.56 z^{-2}}
\]

where \( y_k \) is the feedstock quality at the exit of the tank and \( w_k \) is random white noise.

• RON & MON for each feedstock have a positive covariance structure.
• Simulation parameters:
  • Run-length = 100 days
  • RTO execution frequency = 12/day

• Four algorithms are used in turn to control the blending process.
  • LP + bias (Conventional industrial controller)
  • Dynamic blend-over-horizon controller (Singh, 1996)
  • Static CCP
  • Dynamic CCP (Proposed controller)
Infeasible blends = 70

- The blends that violate RON and MON constraints are very close to specs.
- However, the RVP violations are relatively bigger.
Infeasible blends = 78

- Almost all infeasible blends are due to RVP violations which are largely negligible in magnitude.
Results III

- The back-offs are very large for all qualities.
- Only RVP specs are violated, and the magnitude of violation is high.

Infeasible blends = 23
Infeasible blends = 7

• The constraint violations can hardly be spotted!
Conclusions

- Static Chance Constrained Programming is a poor choice especially if the disturbances are non-stationary.
- RTO with “bias” updating works to a fair degree with non-stationary disturbances because it is designed for step disturbances. However for our case, too many infeasible solutions were generated.
- Time-horizon controller gives pretty good results in our case.
- Dynamic CCP gives very good results as it tracks the disturbances well and generates “optimal” back-offs from the uncertain constraints. The combination of probabilistic programming with the dynamic controller is very promising.
Future work

- Need to work on developing algorithms which solve the Probabilistic Programming problem with joint probabilistic constraints without using approximate probabilities.
- Need to look at the effect of structural model mismatch
  - true model is non-linear.
  - performance with an identified linear model needs to be looked into.
- The non-linear Chance Constrained problem is still unsolved.
  - In our case performance was examined with CCP applied to a linearized model.
- Methods are needed to incorporate uncertainty in the objective function parameters ($c$).
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