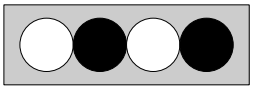
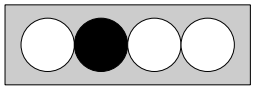
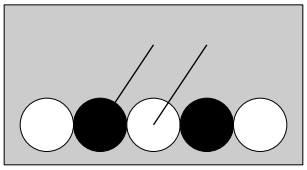
cmput 670 2026 homework 3

1. Recall $\uparrow = \{0|*\}$. Let $g = \uparrow *$. (a) Prove $g = \{*, \uparrow | 0, \uparrow\}$. (b) Prove $g = \{*, 0|0\}$.

   Recall $\Uparrow = \uparrow + \uparrow$. (c) Prove $\Uparrow = \{\uparrow \mid \uparrow *\}$. (d) Prove $\Uparrow = \{0| \uparrow *\}$.

2. Let $\mathcal{O} = \{\ \phi,\ 0,\ 1,\ -1,\ *,\ \{0, *\}\ \}$. For each S in $\mathcal{O}$ and each T in $\mathcal{O}$, give the canonical form of $g = \{S|T\}$. Show your work (answers on the next page).

3. Prove the canonical forms of clobber games `oxox=ox4`, `oxoo`, `oxoxo=ox5` are respectively $\pm\{*, \uparrow\}$, $\uparrow * = \{0, *|0\}$, $\{\uparrow | *\}$. For `ox5`, the diagram below shows the two canonical moves.

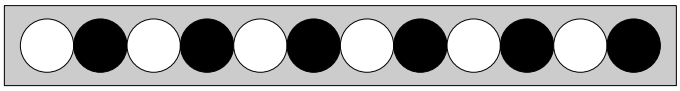




4. a) Give the number of move options for Left from `oxoxoxoxoxox=ox12`.

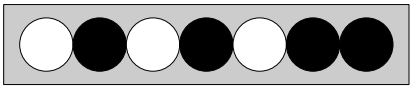
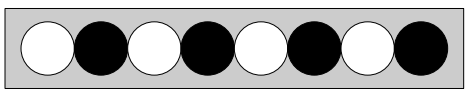
   b) Recall: for any game g, $g*$ is defined as $g + *$. Recall: the canonical form of `ox4*` is $\pm\{0, \uparrow *\}$. Assume in a clobber game that Left plays on a part (connected subgame) `ox12`: `ox12` $\equiv$ `ox4*`, so there are only two moves from `ox12` that Left needs to consider: what are they? Write each of your two answers like this:

   L plays to the left on `ox12` leaving `oxx xoxoxoxo`, or
   L plays to the right on `ox12` leaving `oxoxoxoxo xx`.

   Explain how you can use CGSuite to check your answer.



5. (a) Prove $\downarrow * <$ `oxoxoxox=x8`.
   (b) Prove the canonical form of `ox8` is $\pm\{$ `ox5`, `ox4` $\uparrow\}$. Hint: you might find this useful: the canonical form of `xxoxoxo=xx7` is $\{\Uparrow \mid \downarrow *, \pm\{0, \uparrow *\}\}$.

1. answers

	ϕ	0	-1	1	$*$	$\{0, *\}$
ϕ	0	?	?	?	?	?
0	1	$*$	?	?	?	?
1	2	$\{1 \mid 0\}$	± 1	?	?	?
-1	0	$-\frac{1}{2}$	$-1*$	0	?	?
$*$	0	$\downarrow$	$\{*\mid -1\}$	0	0	?
$\{0, *\}$	1	$\uparrow *$	$\{0, *\mid -1\}$	$\frac{1}{2}$	$\uparrow$	$*2$