cmput 670 2026 homework 1

1. a) Define combinatorial game.
   b) Give two reasons bridge is not a CG.
   c) Give two reasons rock-paper-scissors is not a CG.
   d) A CG is called *normal play* if ... (finish the sentence).
   e) A CG is called *win/loss* if ...
   f) A CG is called *win/loss/draw* if ...
   g) For a player in a CG, a strategy is called *winning* if ...
   h) For a player in a CG, a strategy is called *drawing* if ...

2. a) Give the rule for making a legal move in Go.
   b) Is Go a CG? If no, why not? If yes, why is the game finite?

3. Recall: a CG is *win/loss* if each terminal game position is a win or a loss for the player-to-move. Prove Zermelo's theorem for win/loss CGs:
   a) Give the statement that you will prove.
   b) Argue by induction: prove the statement for the base case(s).
   c) Continue the PBI: what is the inductive assumption that you will make?
   d) Continue the PBI: make the inductive assumption and finish the proof.

4. a) For a CG, who are Left and Right?
   b) For a CG, define outcome classes $\mathcal{P}$, $\mathcal{N}$, $\mathcal{L}$, and $\mathcal{R}$.

5. a) Give the outcome class for each of these clobber games: `oxoxox`, `oxoxo`, `ooxo`, `oox`.
   b) For clobber game `xxox`, give i) the top two levels of the tree of all continuations ii) the game notation for g.

6. a) Prove that $n\times n$ hex is in class $\mathcal{N}$.
   b) Can the strategy-stealing argument used to prove that $n\times n$ hex is in $\mathcal{N}$ be used to prove that, for 19×19 go with komi 0, Black has a drawing strategy?
   c) Repeat b) for komi 0.5.

7. a) Which of the following nim games are first-player-win? Show your work.
   (11,12,13) (11,13,24) (12,13,14)
   b) For nim (11,12,13,14), find all winning moves. Show your work.

8. a) For CG games G and H, define $G + H$.

   b) Recall that 0 is the game $\{|\}$. Prove that $G + 0 = G$.

   c) For CG game G, define $-G$.

   d) CG games G and H are defined as *equivalent* (written $G \equiv H$) if $G + -H$ is a $\mathcal{P}$-position. Let P be any $\mathcal{P}$-position. Prove that $P \equiv 0$.

9. Prove that $\uparrow * \equiv \{0, *|0\}$.

10. Below are the top 3 levels of the game tree (tree of all continuations) of 2x2 go, with symmetry pruning. Each pass move is shown as an empty circle. Draw the next two levels of the game tree (you can prune for symmetry).