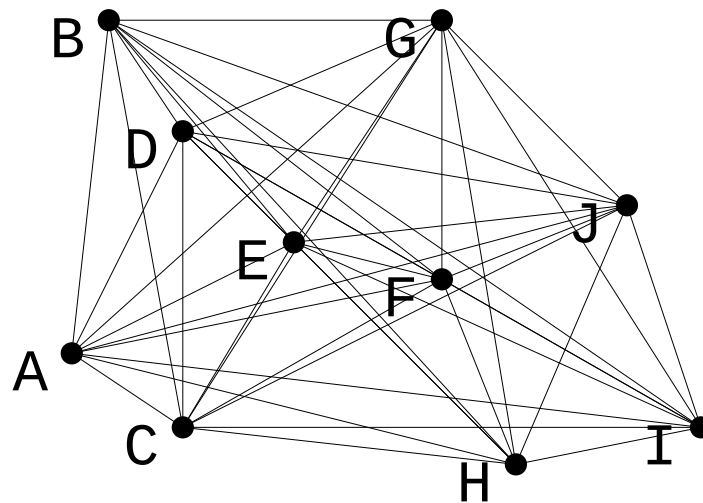


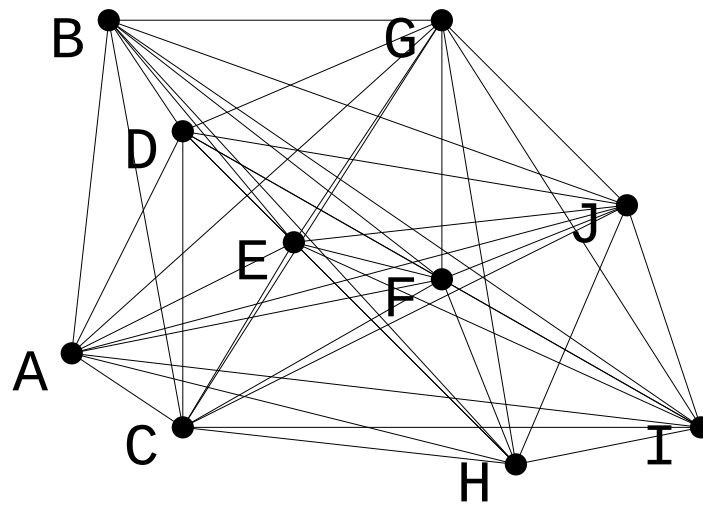


tsp 2-approx algorithm

- * duplicate MST
- * take 2-MST Eulerian tour
- * shortcut tour (skip repeated nodes)
 => Hamiltonian cycle MSTHC

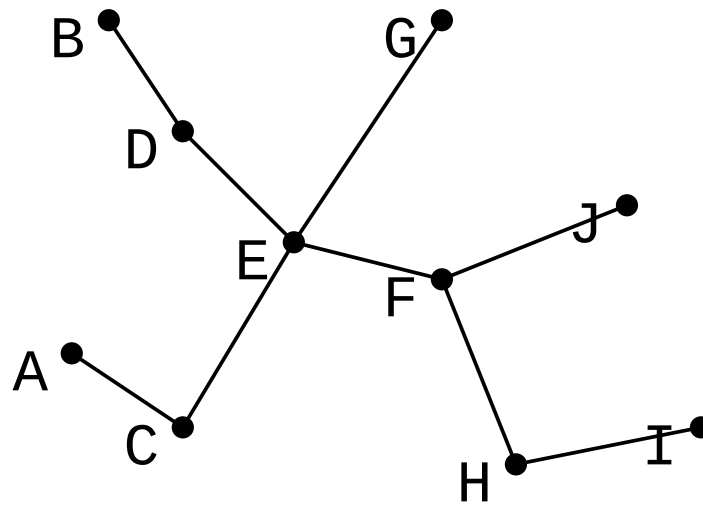
correctness

- * $\text{wt}(\text{MSTHC}) \leq 2 \cdot \text{wt}(\text{opt cycle})$



	A	B	C	D	E	F	G	H	I	J
A		5	2	4	4	6	7	7	9	9
B			6	2	4	6	5	9	11	8
C				4	3	4	7	5	8	7
D					2	4	4	7	9	7
E						2	4	5	7	5
F							4	3	4	3
G								7	7	4
H									3	4
I										4

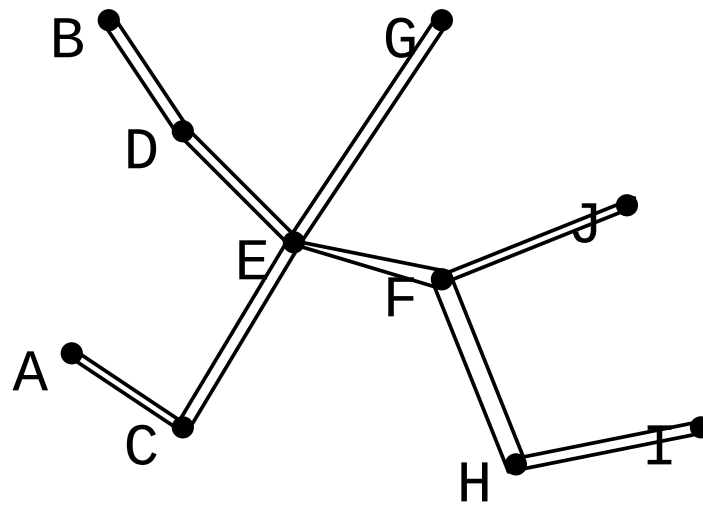
input: complete weighted graph



	A	B	C	D	E	F	G	H	I	J
A		5	2	4	4	6	7	7	9	9
B			6	2	4	6	5	9	11	8
C				4	3	4	7	5	8	7
D					2	4	4	7	9	7
E						2	4	5	7	5
F							4	3	4	3
G								7	7	4
H									3	4
I										4

find an MST

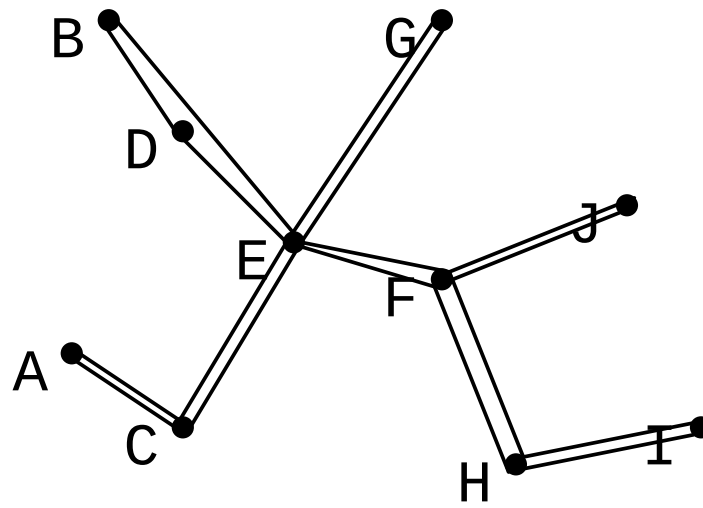
weight 24



initial Hamiltonian tour:

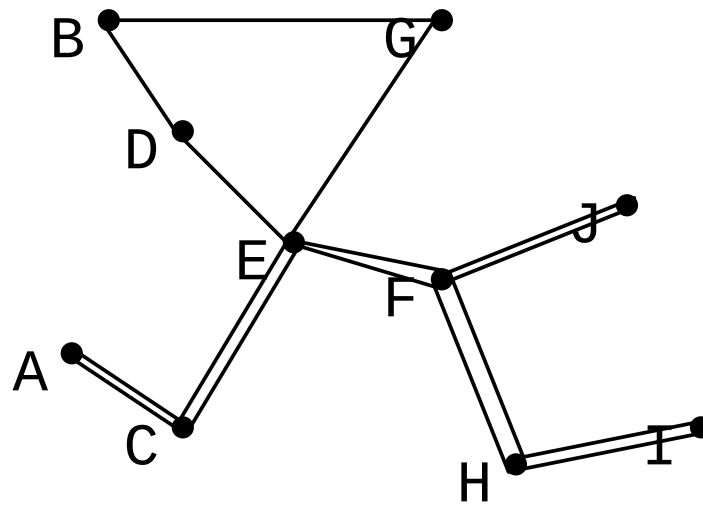
any duplicate-MST-Eulerian-tour

A C E D B D E G E F J F H I H F E C A



1st shortcut: skip repeated D

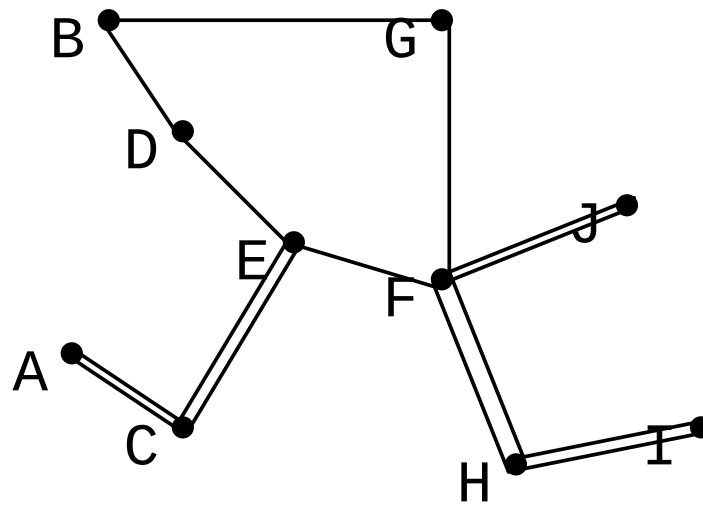
A C E D B E G E F J F H I H F E C A



2nd shortcut: skip repeated E

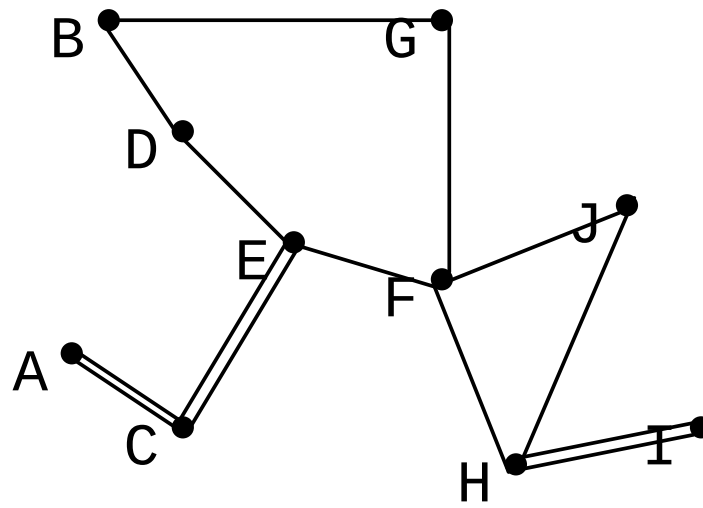
A C E D B

G E F J F H I H F E C A



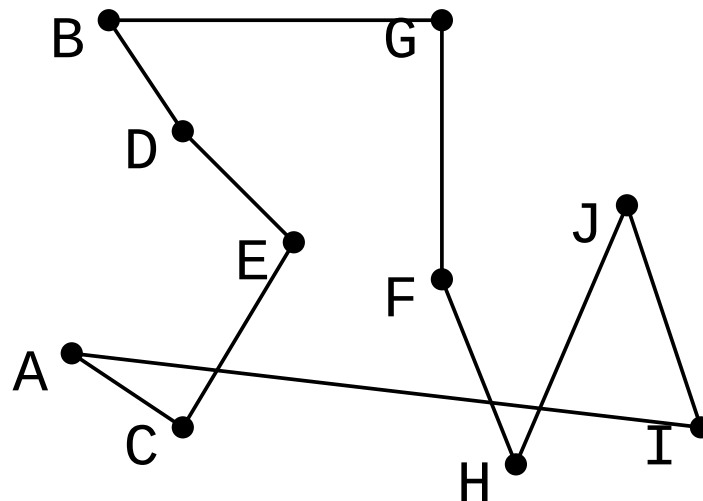
3rd shortcut: skip repeated E

A C E D B G F J F H I H F E C A



4th shortcut: skip repeated F

A C E D B G F J F H I H F E C A



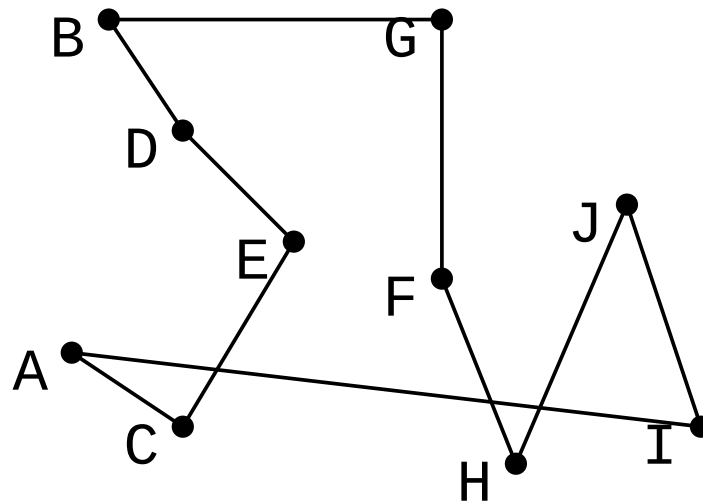
9th shortcut: skip repeated C

A C E D B G F H H I A

done! tour => cycle: no repeated node :)

this Hamiltonian cycle is

at-most-2 approx



(option: uncross each crossing)

uncross IA CE => IC AE

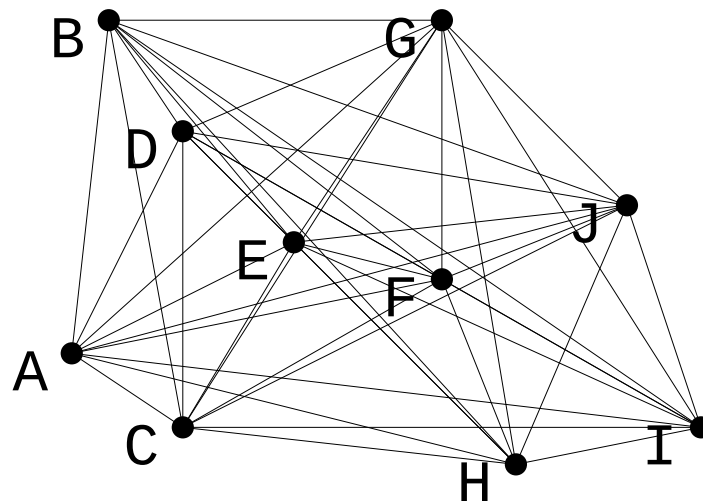
A	C	E	D	B	G	F	H	J	I		A
A		E	D	B	G	F	H	J	I	C	A

FH IC => FI HC

A		E	D	B	G	F	H	J	I	C	A
A		E	D	B	G	F	I	J	H	C	A

FI JH => FJ IH

A		E	D	B	G	F	I	J	H	C	A
A		E	D	B	G	F	J	I	H	C	A

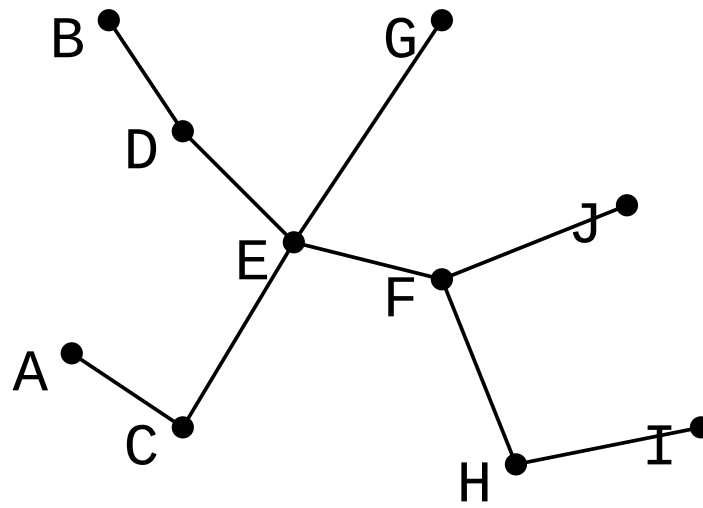


tsp 1.5-approx algorithm

- * MST
- * M: min matching on odd-degree-nodes of MST
- * shortcut MST + M
 => Hamiltonian cycle MSTMHC

correctness

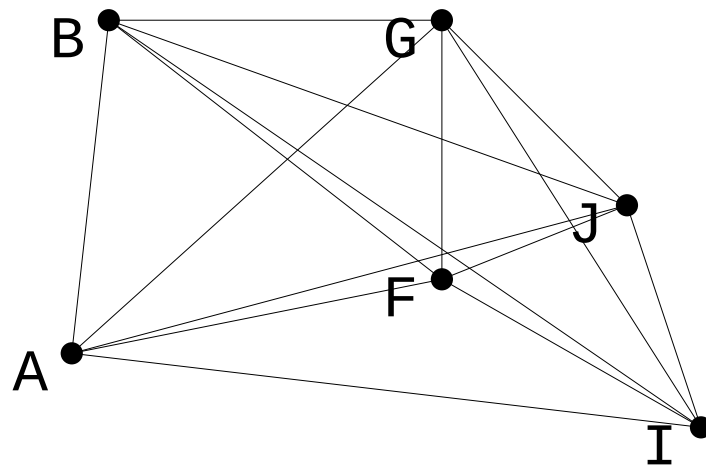
- * $\text{wt}(\text{MSTMHC}) \leq 1.5 \cdot \text{wt}(\text{opt cycle})$



	A	B	C	D	E	F	G	H	I	J
A		5	2	4	4	6	7	7	9	9
B			6	2	4	6	5	9	11	8
C				4	3	4	7	5	8	7
D					2	4	4	7	9	7
E						2	4	5	7	5
F							4	3	4	3
G								7	7	4
H									3	4
I										4

find an MST

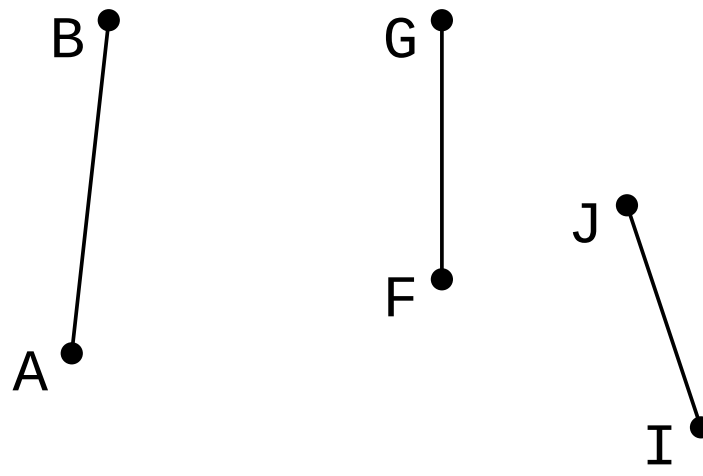
weight 24



complete subgraph induced by MST-odd-deg nodes:

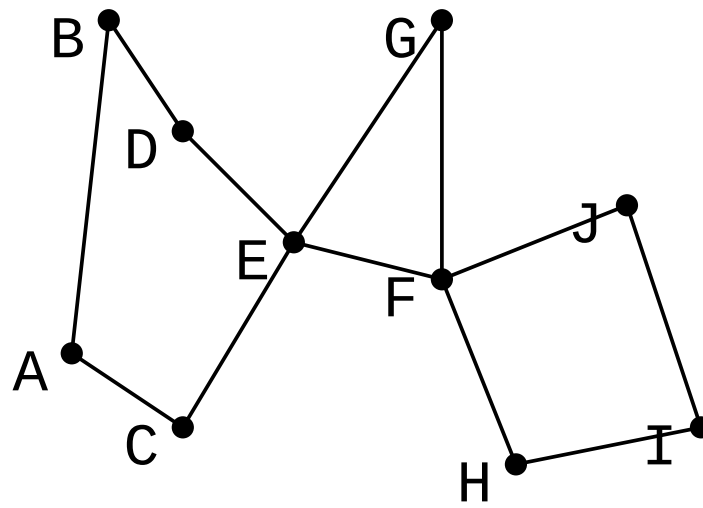
find min weight perfect matching M

	A	B		F	G		I	J
A		5		6	7		9	9
B				6	5		11	8
F					4		4	3
G							7	4
I								4



	A	B		F	G		I	J
A		5		6	7		9	9
B				6	5		11	8
F					4		4	3
G							7	4
I								4

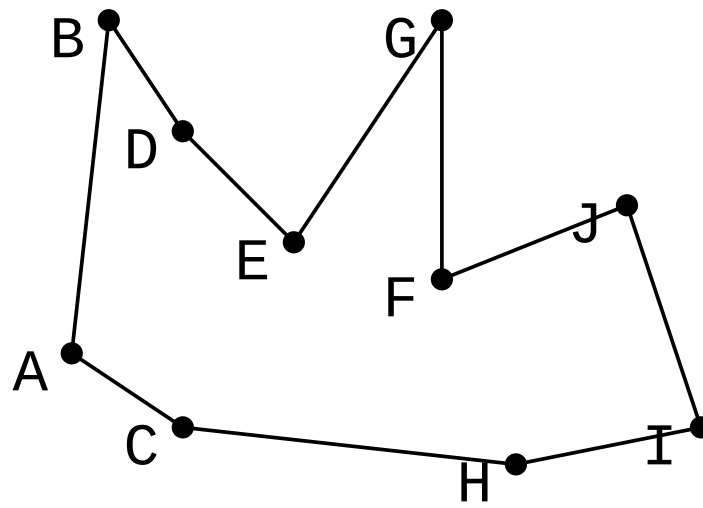
min-weight perfect matching M wt 13



	A	B	C	D	E	F	G	H	I	J
A		5	2	4	4	6	7	7	9	9
B			6	2	4	6	5	9	11	8
C				4	3	4	7	5	8	7
D					2	4	4	7	9	7
E						2	4	5	7	5
F							4	3	4	3
G								7	7	4
H									3	4
I										4

MST + M

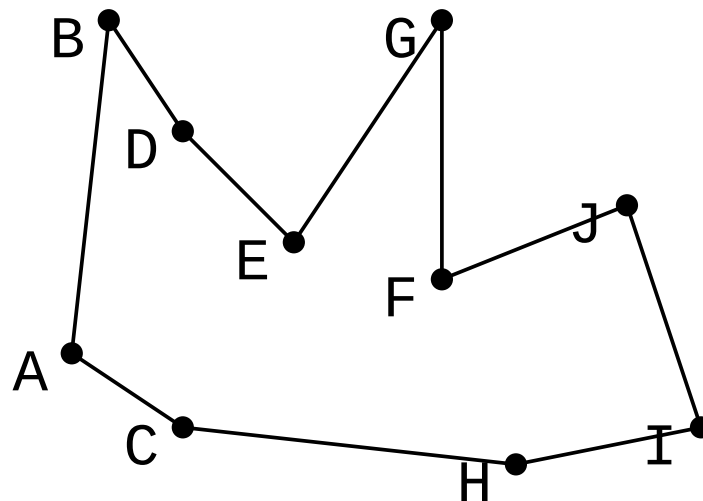
wt 37



	A	B	C	D	E	F	G	H	I	J
A		5	2	4	4	6	7	7	9	9
B			6	2	4	6	5	9	11	8
C				4	3	4	7	5	8	7
D					2	4	4	7	9	7
E						2	4	5	7	5
F							4	3	4	3
G								7	7	4
H									3	4
I										4

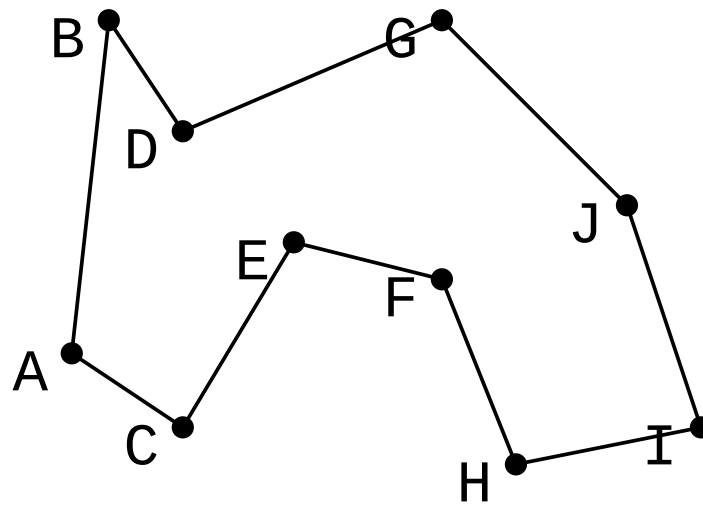
shortcut MST+M

34



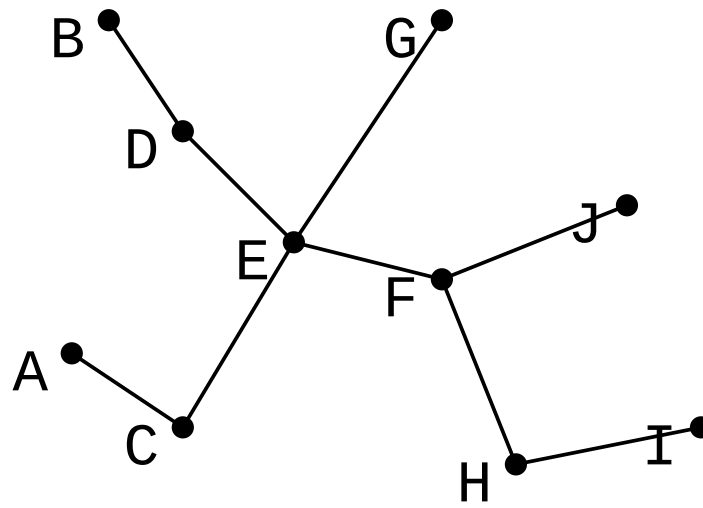
correctness

- * $\text{wt}(\text{MST}) \leq \text{wt}(\text{graph Hamiltonian cycle})$ why?
: opt-cycle minus edge is spanning tree
- * $\text{wt}(\text{MST-odd-deg Ham-cycle}) \leq \text{wt}(\text{graph Ham-cycle})$
: delete nodes, shortcut
- * $\text{wt}(\text{MSTod min-wt perf-match}) \leq \text{wt}(\text{MSTod Ham-cycle})/2$
: MSTod Ham-cycle partitions into M1 and M2
: min of these has $\text{wt } m_0 \leq \text{wt}(\text{MSTod Ham-cycle})/2$
: $\text{wt}(\text{MSTod min-wt perf-match}) \leq m_0$



	A	B	C	D	E	F	G	H	I	J
A		5	2	4	4	6	7	7	9	9
B			6	2	4	6	5	9	11	8
C				4	3	4	7	5	8	7
D					2	4	4	7	9	7
E						2	4	5	7	5
F							4	3	4	3
G								7	7	4
H									3	4
I										4

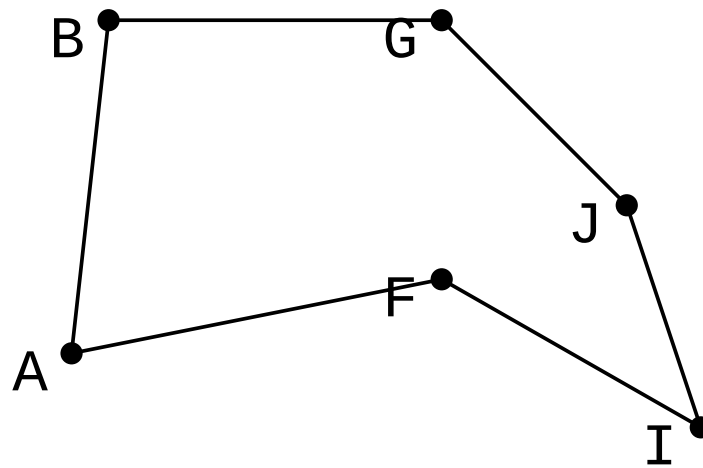
optimal cycle (brute force) 32



an MST

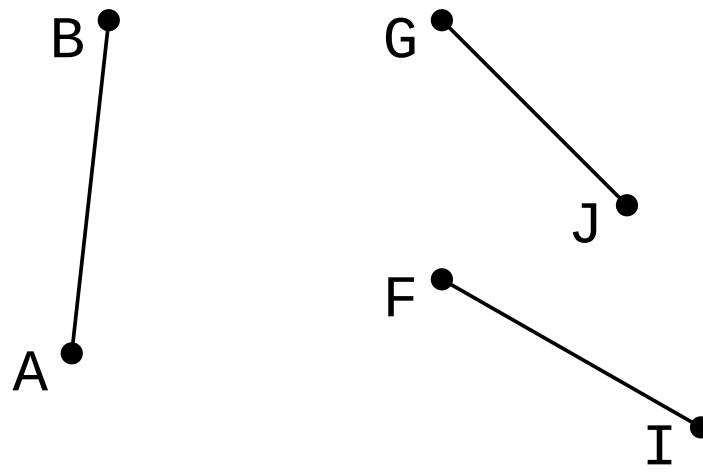
weight 24

odd-degree nodes A B E G J I



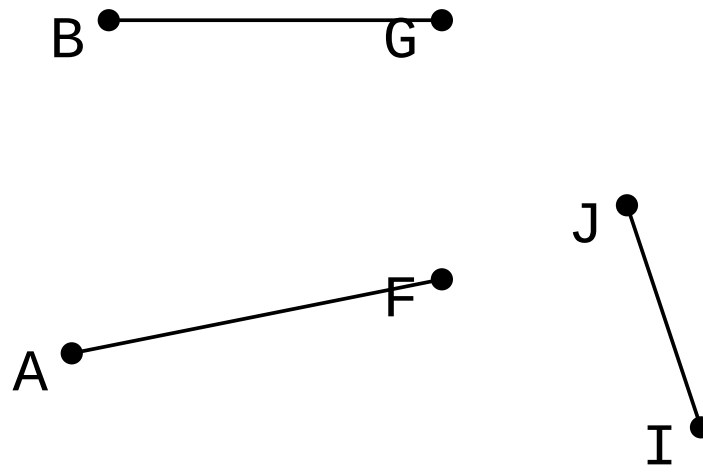
	A	B		F	G		I	J
A		5		6	7		9	9
B				6	5		11	8
F					4		4	3
G							7	4
I								4

MST-odd-deg-nodes: opt-cycle wt 28



	A	B		F	G		I	J
A		5		6	7		9	9
B				6	5		11	8
F					4		4	3
G							7	4
I								4

MST-odd-deg-nodes: opt-cycle, matching 1, wt 13



	A	B		F	G		I	J
A		5		6	7		9	9
B				6	5		11	8
F					4		4	3
G							7	4
I								4

MST-odd-deg-nodes: opt-cycle, matching 2, wt 15