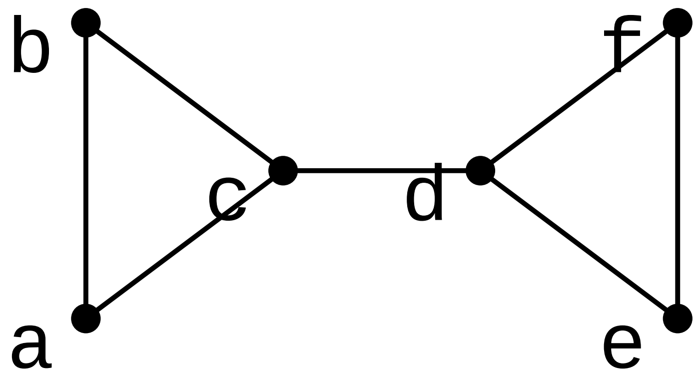


min cut

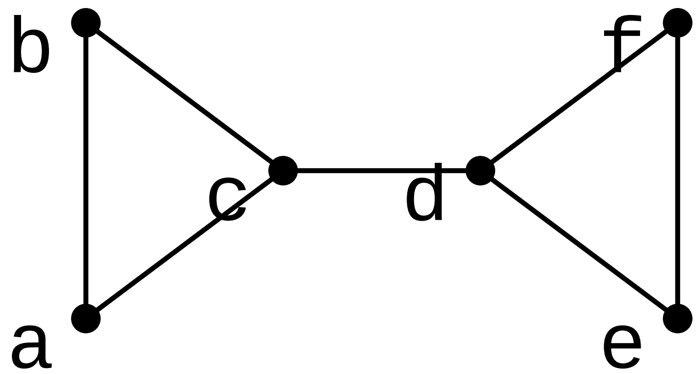
* unweighted graph G, n nodes

* cut: bipartition of node set
    e.g. {a,d} {b,c,e,f}

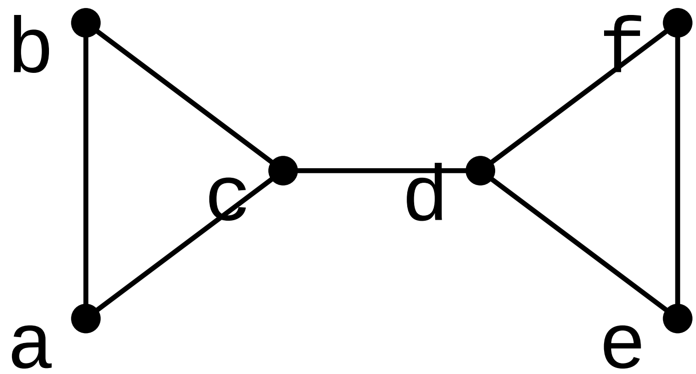
* an edge *crosses* a cut A,B
    if it has one end in each part
    e.g. {a,b}, {a,c}, {c,d}, {d,f}, {d,e}

* cut size: number of edges that cross it
    e.g. 5

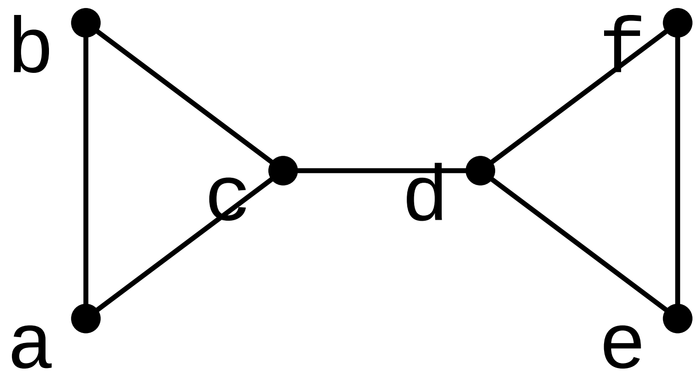
* min cut problem: given G, find any min cut



brute force min cut alg'm ?



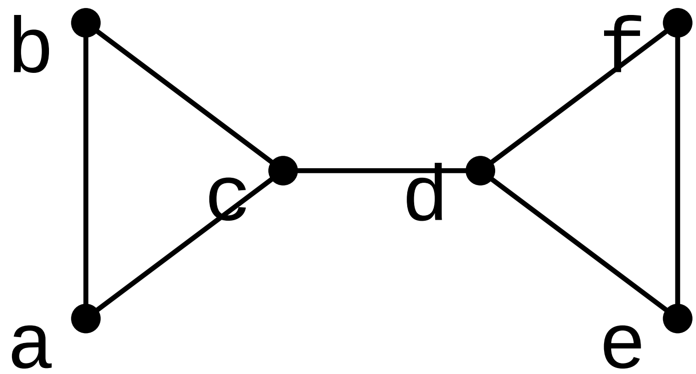

```
brute force min cut alg'm

   - try all possible cuts

     e.g.  {a} {bcdef}
           {ab} {cdef}
           {ac} {bdef}
               ...
           {abcde} {f}

   total number of cuts, n nodes =
(number subsets of n-1 set) - 1 = 2^{n-1} - 1

  [ bipartition defined by making left part
    element a plus each partial subset      ]

     here: 2^{n-1} - 1 = 2^{5} - 1 = 31
```



randomized kruskal min cut

* unweighted graph, n nodes
* process edges in uniform-random order
* apply Kruskal MST until exactly 2 components

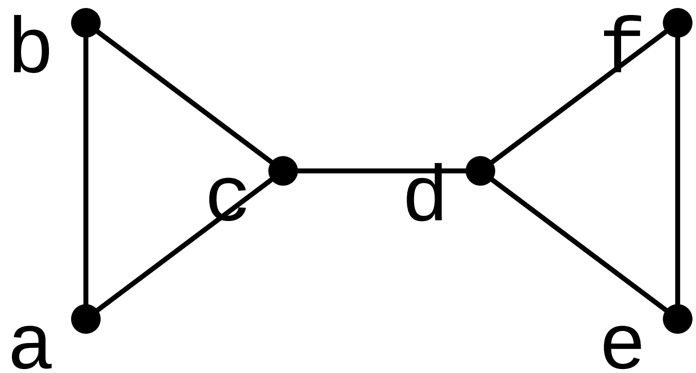
```
edges (random order)  -  bc ef ac ab cd de ef
number components:    6   5  4  3  3  2

cut  bipartition {a b c d} {e f}

cut  cross-edges {de, df}

cut size         2
```



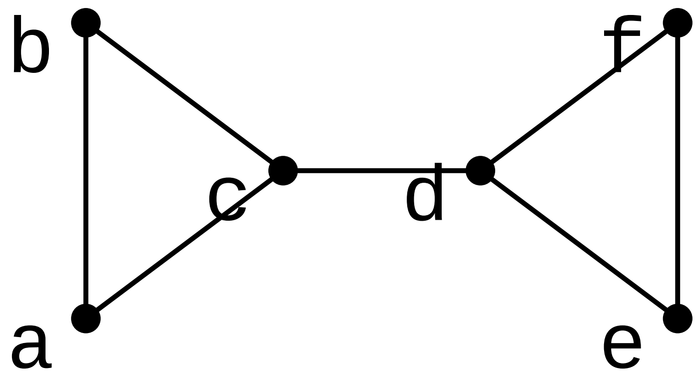
randomized kruskal min cut

* unweighted graph, n nodes
* consider edges in uniform-random order
* apply Kruskal MST until exactly 2 components
* prob(this cut X is a min cut) >= 2/(n(n-1)) (1)

proof of (1) ?



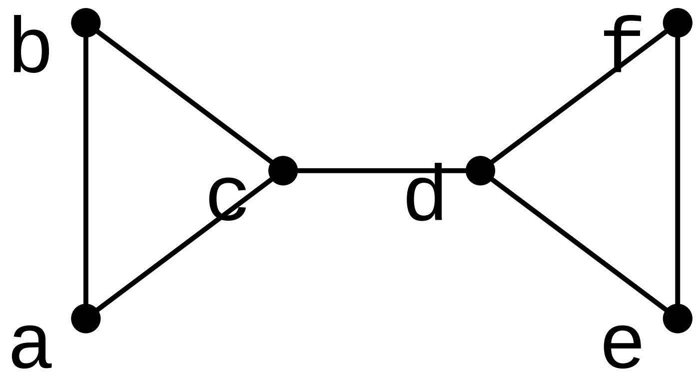

```
above example

  * prob(RKMC cut is min) >= 2/(6*5) = 1/15 = .0666...

  * prob(                ) =  ?
      * unique min cut (UMC) {{a, b, c}, {d, e, f}}
      * consider all 7! edge permutations
      * RKMC(permutation) returns UMC iff
          - in perm'n, edge cd is last, or
          - in perm'n, edge cd is 2nd-last, or
          - in perm'n, edge cd is 3rd-last and last
            two edges are from different triangles
      * prob = 1/7 + 1/7 + (1/7)(3/5)     exercise

             = 13/35 = .3714...
```



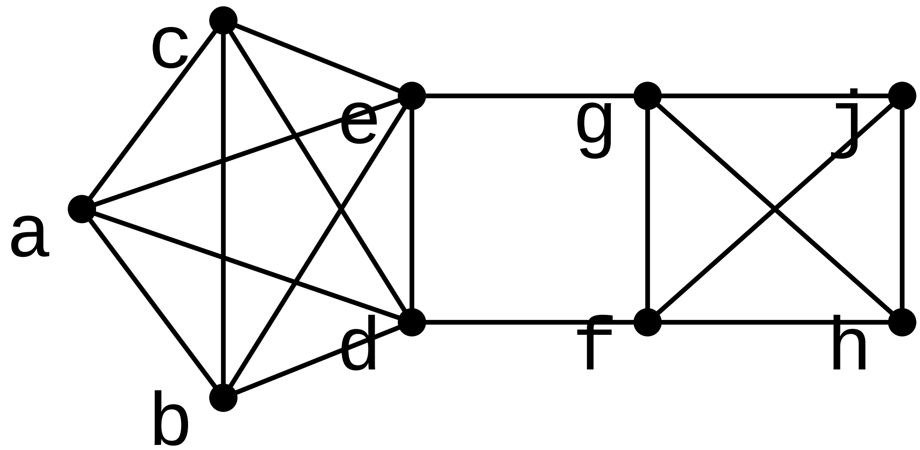
* call RKMC t times, take smallest cut found
* prob(best cut found is min cut) ?

t	1.0 - (1.0 - 13/35)^t	
1	.371	
2	.604	with bound (1)
3	.751	67 trials
4	.843	prob >= .99
5	.901	
6	.938	with n = 100
7	.961	bound (1)
8	.975	22794 trials
9	.984	prob >= .99
10	.990	

after 10 trials
prob(min cut) >= .99




```
proof of (1)

  * F is RKMC forest-so-far
  * after t steps, F has k = n-t components
  * number edges leaving component Cj of F ?
  * consider min cut with cross-edge set Y, |Y| = y
  * { Cj, V - Cj} is a cut, so
      at least y edges leave Cj
  * so number edges between components is ky/2
  * edges between components are exactly the
      RKMC-eligible edges
  * prob(next edge picked is in Y)
      <= y / (ky/2)  =   2/k
  * prob(RKMC picks no edge from Y)
         n-2 n-3 n-4     3 2 1     2
      >= --- --- --- ... - - - = ------
          n  n-1 n-2     5 4 3   n(n-1)
```



exercise