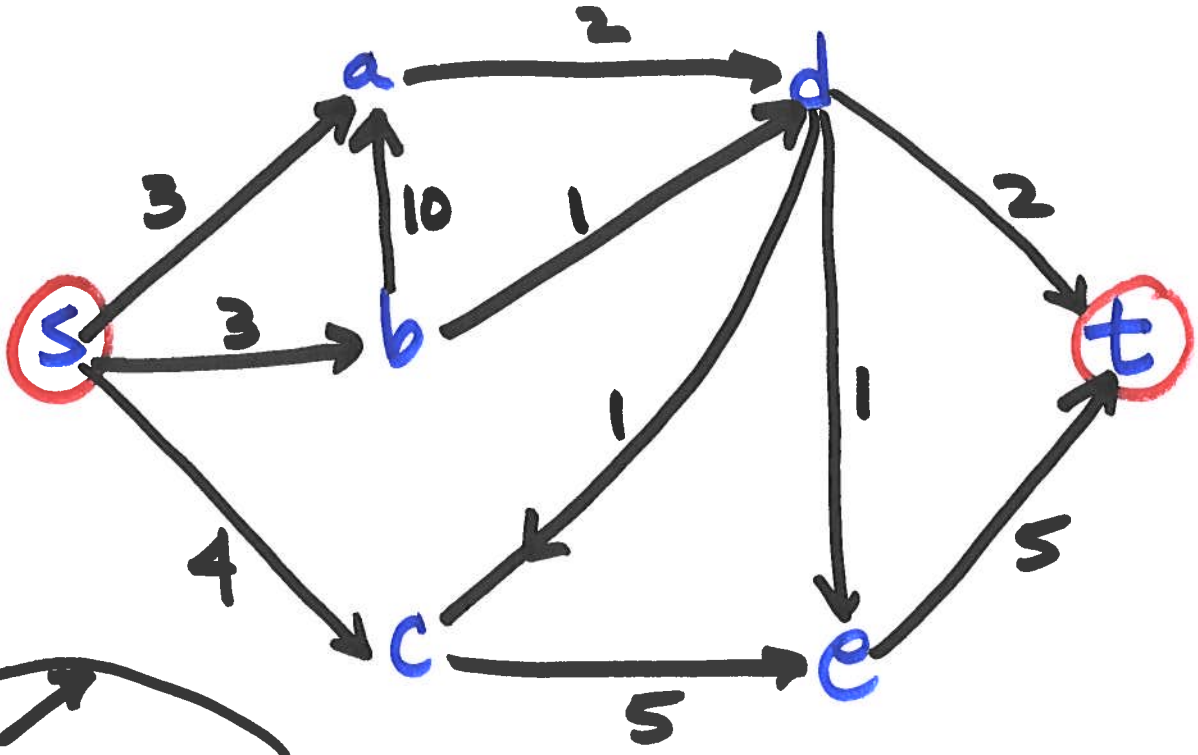
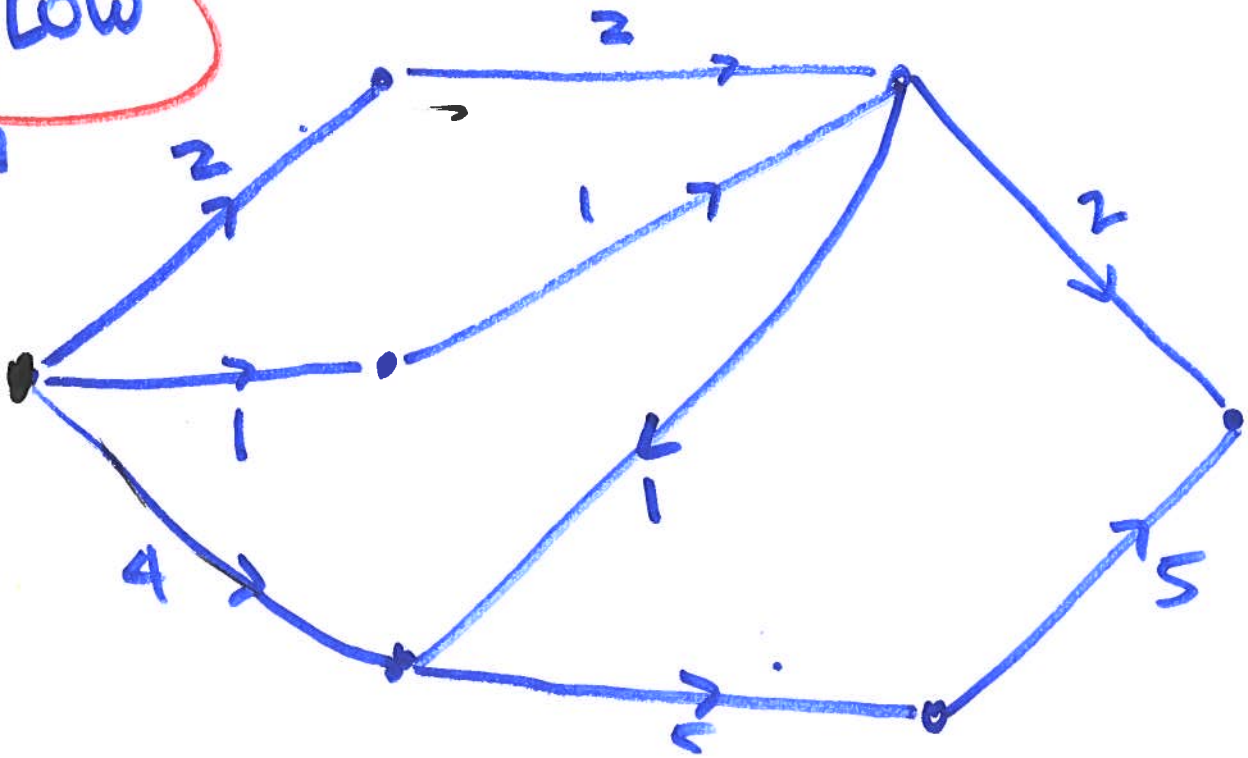
NETWORK FLOW



CAPACITY

A FLOW



MAX FLOW PROBLEM

INST. . DIGRAPH (NETWORK)

- CAPACITIES
- s, t

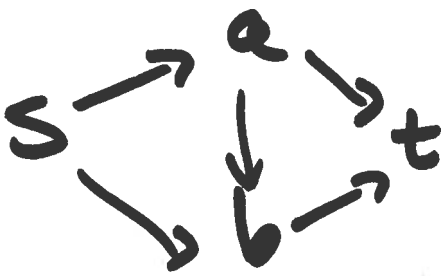
QUERY MAX $s \to t$ FLOW?

FLOW

- FLOW OUT OF s =
- " INTO t
- $\forall$ OTHER NODES v

FLOW INTO v = FLOW OUT OF v

TOY MAX FLOW INST.



ALL CAP. 1

MAX $f_{sa} + f_{sb}$

S.T.

$f_{sa} \leq 1$

$f_{sb} \leq 1$

$f_{ab} \leq 1$

$\vdots$

$f_{bt} \leq 1$

$f_{sa} \geq 0$

$\vdots$

$f_{bt} \geq 0$

- $f_{sa} + f_{sb} = f_{at} + f_{bt}$
- $f_{sa} = f_{ab} + f_{at}$
- $f_{sb} + f_{ab} = f_{bt}$

$f_{\cdot} \in \mathbb{Q}$

- MAX FLOW CAN BE FORMULATED AS LP
- MAX FLOW IS IN $\mathbb{P}$ 

(DECISION)
- YES/NO VERSION OF MAX-FLOW:

  K-FLOW

- INTEGER K-FLOW $\in$ NP-C

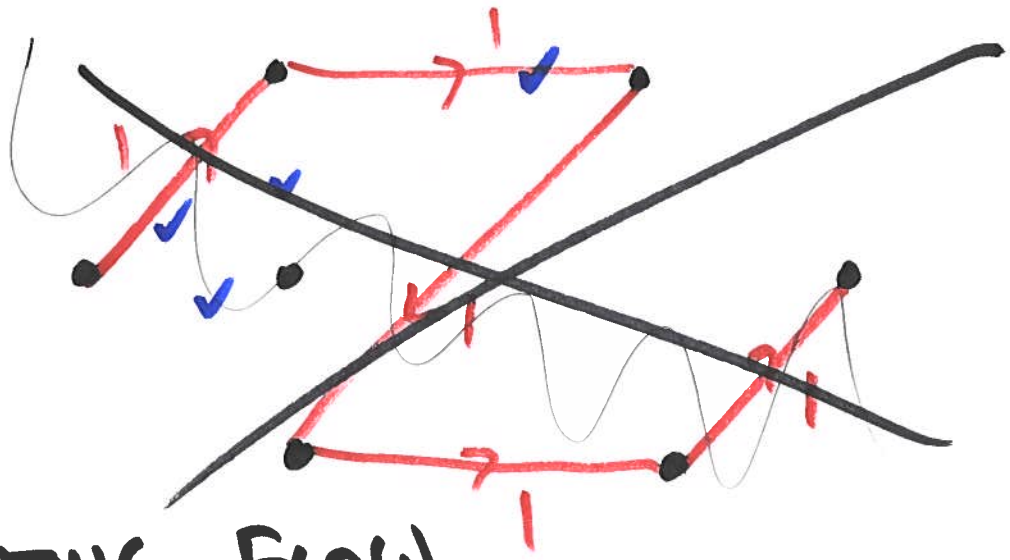
- IF WE LOOK AT HOW SIMPLEX SOLVES MAX-FLOW...

  ⇒ RESIDUAL FLOW ALG'M

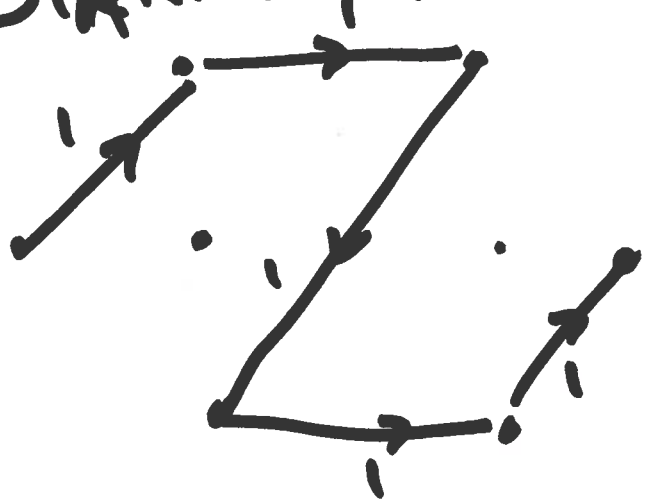
REPEAT:

UPDATE

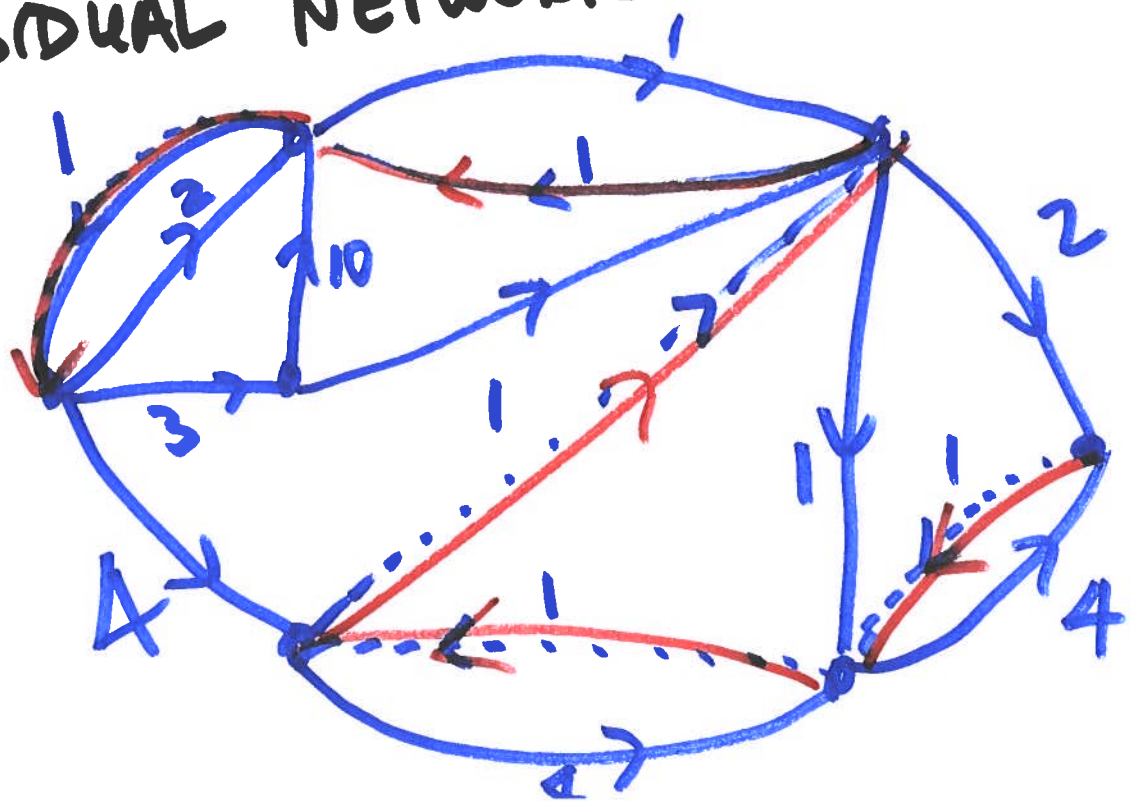
- ~~COMPUTE~~ RESIDUAL NETWORK
- FIND POSITIVE s-t FLOW IN RES. NET



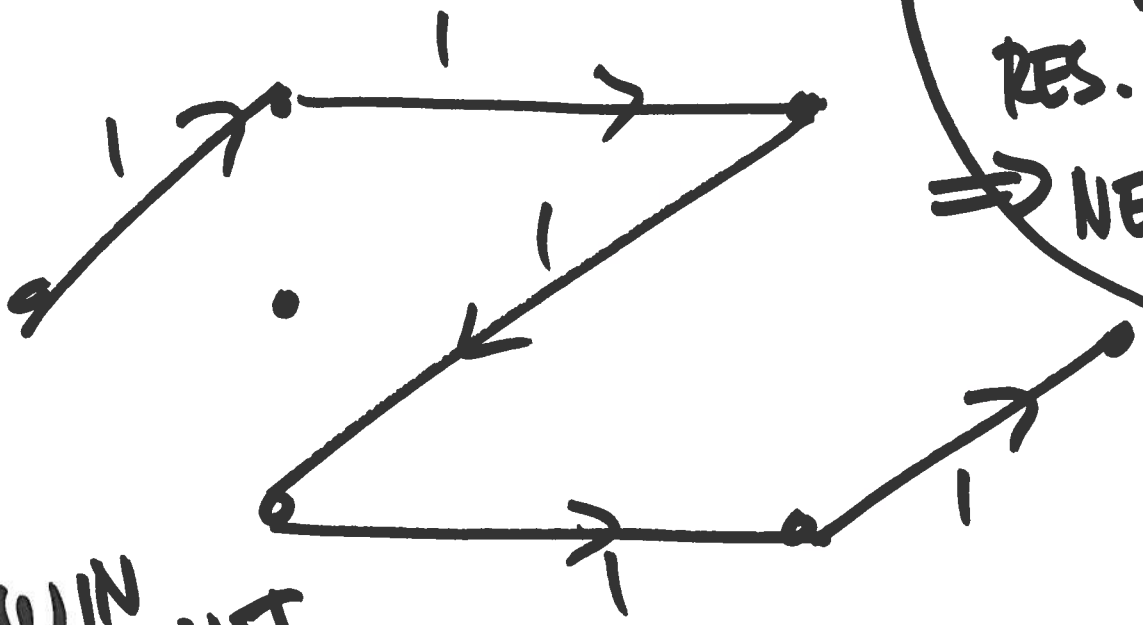
STARTING FLOW



RESIDUAL NETWORK

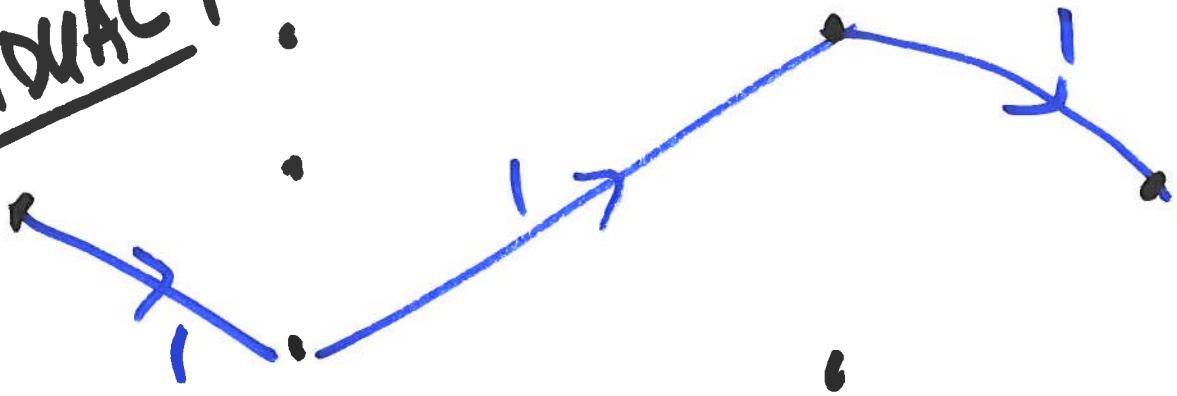


PREV FLOW

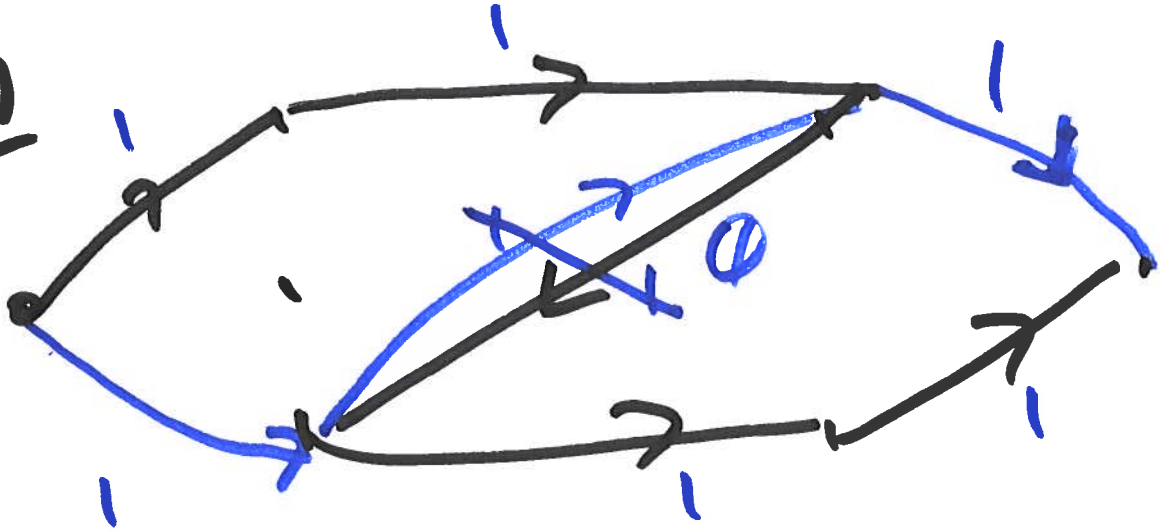


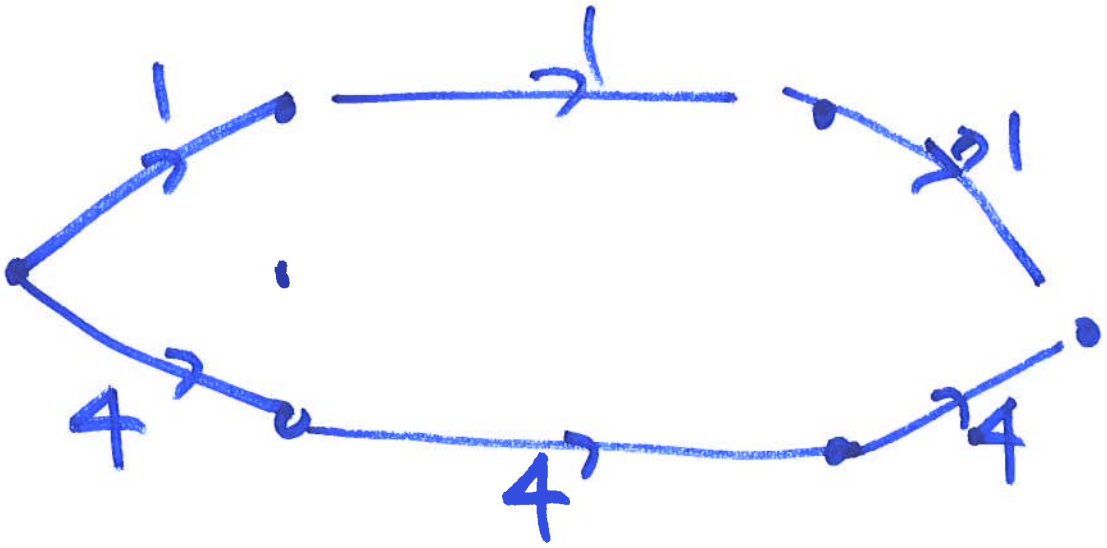
COMBINE
PREV FLOW
+
RES. NET. FLOW
⇒ NEW FLOW

FLOW IN RESIDUAL NET.

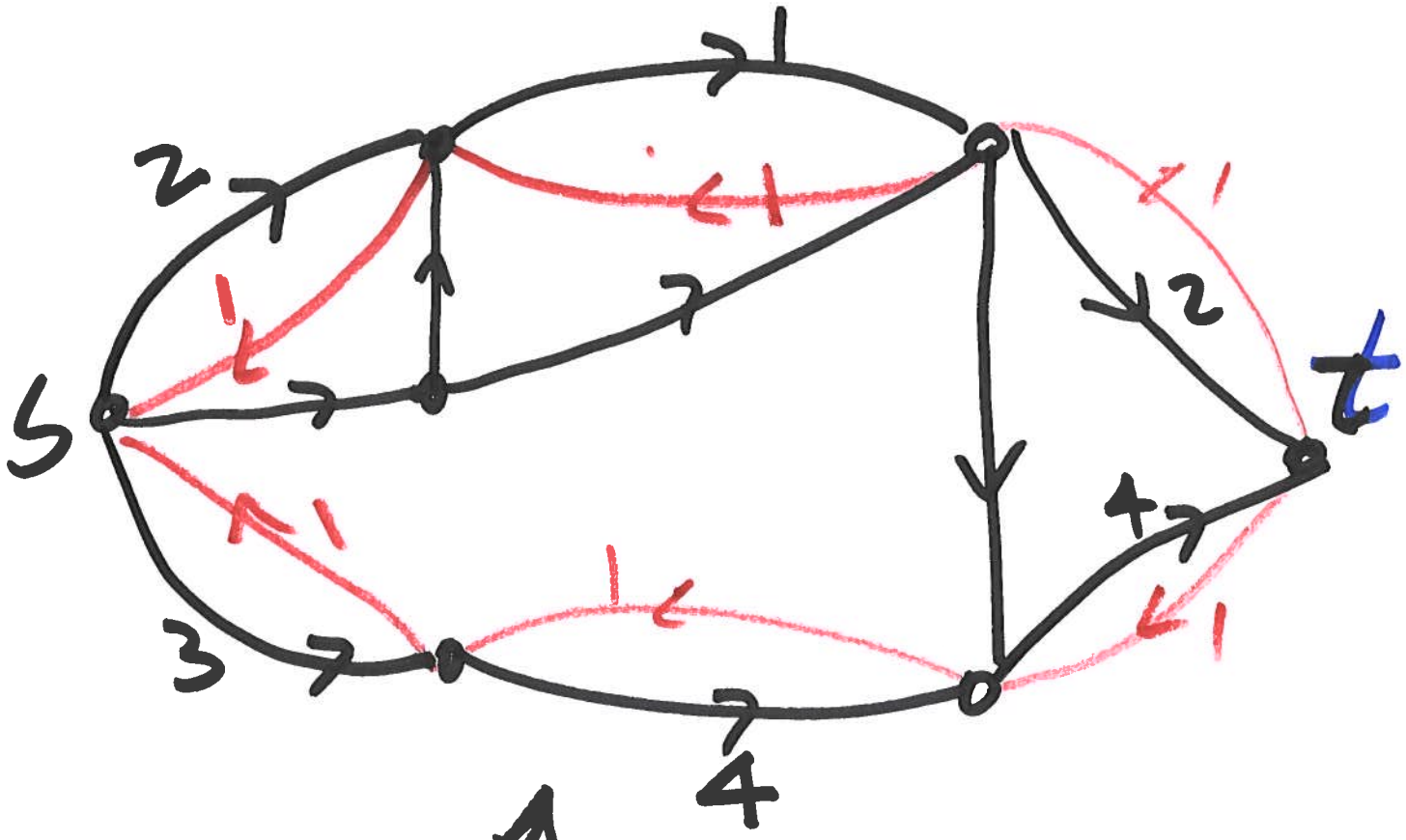


NEW



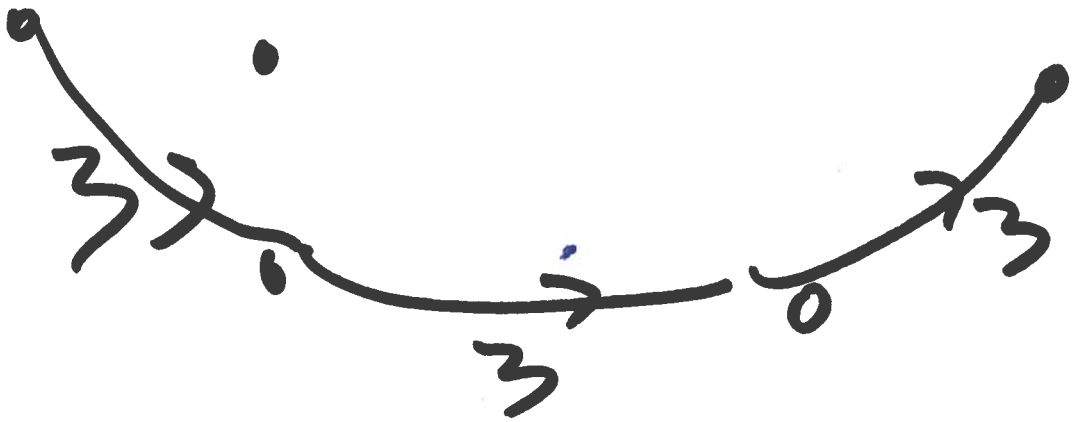


NEW TOTAL FLOW



NEW RESID. NET.

FLOW



ETC. ...

FEASIBLE BOUNDED

LP SOLUTION $\Rightarrow$

$\exists$ DUAL SOLUTION, SAME VALUE } CERTIFICATE OF OPTIMALITY

THE VOLUME OF A MAX FLOW

$=$

THE CAPACITY OF A MIN S-t CUT