family name ______________ given name ______________ ID ______________

quiz1 2013oct4 c204   10marks 30min   no electronic devices

1. [1.5 marks] For each $f(n)$, give the simplest $g(n)$ so that $f(n) = \Theta(g(n))$.

   (i) $16\,n^2 \lg n \;+\; 11\,n^{5/2} \;+\; \dfrac{\sqrt{2}\,n^3}{(\lg n)^7}$ $g(n) =$ ______________

   (ii) $1.6\,n^2(\lg n)^5 \;+\; 5^{\lg n}$ $g(n) =$ ______________

   (iii) $39.7 \displaystyle\sum_{j=1}^{n} j^2 \lg j$ $g(n) =$ ______________

2. [1.5 marks] Find some c so that, for all integers $n \geq 1$, $16\,n \lg n + 31\,n \;\leq\; c\,n^2$.

   $c =$ ______________

   Justification:

   ____

   ____

3. [2 marks] Compute $2^{21} \pmod{13}$. Show your work.

so $2^{21} \pmod{13} =$ ________

```
def f(x,y): # y >= 0
  if y==0:
    return 0
  elif 0== y%2:
    return 2*f(x, y/2)
  else:
    return x + 2*f(x, y/2)
```

4. [2.5 marks] Claim. For all integers x, for all integers $y \geq 0$, `f(x,y)` returns $x * y$.

   Proof by induction on y. In the base case, $y =$ ________, so in the execution of $f(x, y)$, the value returned is ________ , which is equal to $x * y$, so the claim holds. Next consider the inductive case. Assume that the claim holds for all values of y in the set $\{0, \ldots, t\}$. We want to show that the claim holds when $y =$ ________ . First assume y is even. Then the value returned is ____________ , which equals ____________ , which equals ____________ , since by the inductive hypothesis the claim holds when y has the value ________ . So the claim holds in this case. [omit the rest of the proof]

5. [1 marks] Give a recurrence relation for the runtime $T(n)$ of `f(x,y)`, where n is the number of bits in x plus the number of bits in y.

$$
\begin{aligned}
T(n) &= \Theta(1) && \text{when } n = 0 \text{ (i.e. } y = 0) \\
&= O(n) + T(\_\_\_\_\_\_\_\_\_\_\_\_) && \text{when } n \geq 1 \text{ and } y \text{ is even} \\
&= O(\_\_\_\_\_\_\_\_\_\_\_\_) + T(\_\_\_\_\_\_\_\_\_\_\_\_) && \text{when } n \geq 1 \text{ and } y \text{ is odd}
\end{aligned}
$$

6. [1.5 marks] `f(x,y)` is implemented efficiently. Multiplication of a k-bit number by 2 takes $O($________$)$ time, and addition of two at-most-k-bit numbers takes $O($ ________ $)$ time. Assume that x and y each have $n/2$ bits. So, the total number of function calls resulting from `f(x,y)` is $\Theta($ ________ $)$, each call takes $O($ ________ $)$ time, so the runtime of `f(x,y)` is $O($ ________ $)$.

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   (ii) $1.6\,n^2(\lg n)^5 \;+\; 7^{\lg n}$ $\qquad g(n) =$ ________________

   (iii) $39.7\displaystyle\sum_{j=1}^{n} j\lg j$ $\qquad g(n) =$ ________________

2. [1.5 marks] Find some c so that, for all integers $n \geq 1$, $12\,n\lg n + 26\,n \;\leq\; c\,n^2$.

   $c =$ ________________

   Justification:

   ____

   ____

3. [2 marks] Compute 3^{21} (mod 13). Show your work.

   so 3^{21} (mod 13) = ________

```
def f(x,y): # y >= 0
  if y==0:
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    return 2*f(x, y/2)
  else:
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```

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T(n) &= \Theta(1) && \text{when } n = 0 \text{ (i.e. } y = 0) \\
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&= O(\underline{\hspace{3cm}}) + T(\underline{\hspace{3cm}}) && \text{when } n \geq 1 \text{ and } y \text{ is odd}
\end{aligned}
$$

6. [1.5 marks] `f(x,y)` is implemented efficiently. Multiplication of a t-bit number by 2 takes $O($________$)$ time, and addition of two at-most-t-bit numbers takes $O($ ________ $)$ time. Assume that x and y each have $n/2$ bits. So, the total number of function calls resulting from `f(x,y)` is $\Theta($ ________ $)$, each call takes $O($ ________ $)$ time, so the runtime of `f(x,y)` is $O($ ________ $)$.