Unless stated otherwise, variables are integers.

Define α as the golden ratio, i.e. $(1+\sqrt{5})/2$.

Define $f(n)$ as the nth Fibonacci number:

$f(0) = 0$, $f(1) = 1$, and, for all $t \geq 2$, $f(t) = f(t-1) + f(t-2)$.

Define $T(n)$ so that $T(0) = T(1) = 4$ and, for all $n \geq 2$, $T(n) = T(n-1) + T(n-2) + 7$.

1. Prove, for all $n \geq 1$, that $\lg n < n$.
2. Prove that $16n^2 + 99n + 16n \lg n$ is in $\Theta(n^2)$.
3. Prove, for all $n \geq 0$, $f(n) < \alpha^n$.
4. Prove, for all $n \geq 3$, $f(n) > \alpha^{n-2}$.
5. (i) Prove, for all $n \geq 0$, $T(n) = 11f(n+1) - 7$.

   (ii) Prove that $T(n)$ is in $O(\alpha^n)$.

   (iii) Prove that $T(n)$ is in $\Theta(\alpha^n)$.

```
def ff(n):
  L = [0, 1]               // set L[0] to 0,  set L[1] to 1
  for j in range(2,n+1):    # from 2 to n
    L.append( L[j-1]+L[j-2] )
    // invariant: last element of L is L[j]
    // invariant: L[j] = L[j-1]+L[j-2]
  return L[n]
```

6. By induction, prove the first invariant.

   By induction, prove the second invariant.

   By induction, prove that the algorithm is correct.